

Exercise 6.1

Prove the following trigonometric identities:

1. $(1 - \cos^2 A) \operatorname{cosec}^2 A = 1$

Sol:

We know $\sin^2 A + \cos^2 A = 1$

$$\sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A \cdot \operatorname{cosec}^2 A$$

$$\Rightarrow \sin^2 A \cdot \frac{1}{\sin^2 A} = 1 \quad \therefore L.H.S = R.H.S$$

2. $(1 + \cos^2 A) \sin^2 A = 1$

Sol:

We know that $\operatorname{cosec}^2 A - \cot^2 A = 1$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\Rightarrow \operatorname{cosec}^2 A \cdot \sin^2 A = 1$$

$$\frac{1}{\sin^2 A} \cdot \sin^2 A \cdot 1$$

$$1 = 1 \quad L.H.S = R.H.S$$

3. $\tan^2 \theta \cos^2 \theta = 1 - \cos^2 \theta$

Sol:

$$L.H.S \Rightarrow \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \cos^2 \theta = \sin^2 \theta$$

$$R.H.S \Rightarrow 1 - \cos^2 \theta \quad [1 = \sin^2 \theta + \cos^2 \theta]$$

$$\Rightarrow \sin^2 \theta \quad [\therefore \sin^2 \theta = 1 - \cos^2 \theta]$$

$$L.H.S = R.H.S$$

4. $\operatorname{cosec} \theta \sqrt{1 - \cos^2 \theta} = 1$

Sol:

$$L.H.S = \operatorname{cosec} \theta \sqrt{\sin^2 \theta} \quad [\therefore 1 - \cos^2 \theta = \sin^2 \theta]$$

$$= \operatorname{cosec} \theta \cdot \sin \theta$$

$$= 1$$

$$\therefore L.H.S = R.H.S$$

5. $(\sec^2 \theta - 1)(\operatorname{cosec}^2 \theta - 1) = 1$

Sol:

We know that $\sec^2 \theta - \tan^2 \theta = 1$

$$\Rightarrow \sec^2 \theta = 1 + \tan^2 \theta$$

$$\operatorname{cosec}^2 \theta - \cos^2 \theta = 1$$

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

$$\tan^2 \theta \cdot \cot^2 \theta = \tan^2 \theta \frac{1}{\tan^2 \theta}$$

6. $\tan \theta \frac{1}{\tan \theta} = \sec \theta \operatorname{cosec} \theta$

Sol:

$$LHS = \tan \theta + \frac{1}{\tan \theta} \Rightarrow \frac{\sin \theta}{\cos \theta} + \frac{1}{\frac{\sin \theta}{\cos \theta}}$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$\Rightarrow \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta}$$

$$\Rightarrow \sec \theta \operatorname{cosec} \theta$$

Hence L.H.S = R.H.S

7. $\frac{\cos \theta}{1 - \sin \theta} = \frac{1 + \sin \theta}{\cos \theta}$

Sol:

$$\cos \theta - \cos 2 \cdot \frac{\theta}{2} = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}$$

$$\sin \theta = \sin \frac{\theta}{2} \cdot 2 \Rightarrow 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$\Rightarrow LHS = \frac{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}{\left[\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} \right] - 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \quad \left[\because 1 = \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} \right]$$

$$\Rightarrow \frac{\left[\cos \frac{\theta}{2} - \sin \frac{\theta}{2} \right] \left[\cos \frac{\theta}{2} + \sin \frac{\theta}{2} \right]}{\left[\cos \frac{\theta}{2} - \sin \frac{\theta}{2} \right]} \quad \left[\because a^2 - b^2 = (a - b)(a + b)(a - b)^2 = a^2 + b^2 - 2ab \right]$$

$$\Rightarrow \frac{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}}.$$

$$R.H.S \frac{1 + \sin \theta}{\cos \theta} \Rightarrow \frac{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} + 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}$$

$$\Rightarrow \frac{\left[\cos \frac{\theta}{2} + \sin \frac{\theta}{2} \right]}{\left[\cos \frac{\theta}{2} - \sin \frac{\theta}{2} \right]}$$

$$\Rightarrow \frac{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}}$$

$$\therefore L.H.S = R.H.S$$

8. $\frac{\cos \theta}{1 + \sin \theta} = \frac{1 - \sin \theta}{\cos \theta}$

Sol:

$$\cos \theta = \cos 2 \frac{\theta}{2} = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}$$

$$\sin \theta = \sin 2 \cdot \frac{\theta}{2} = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$1 = \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2}$$

$$LHS = \frac{\cos \theta}{1 + \sin \theta} = \frac{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}{\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} + 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}$$

$$\Rightarrow \frac{\left(\cos \frac{\theta}{2} - \sin \frac{\theta}{2} \right) \left(\cos \frac{\theta}{2} + \sin \frac{\theta}{2} \right)}{\left(\cos \frac{\theta}{2} + \sin \frac{\theta}{2} \right)^2}$$

$$\Rightarrow \frac{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}$$



$$RHS = \frac{1 - \sin \theta}{\cos \theta} = \frac{\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} - 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}$$

$$\Rightarrow \frac{\left(\cos \frac{\theta}{2} - \sin \frac{\theta}{2} \right)^2}{\left(\cos \frac{\theta}{2} - \sin \frac{\theta}{2} \right) \left(\cos \frac{\theta}{2} + \sin \frac{\theta}{2} \right)}$$

$$\Rightarrow \frac{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}$$

$$\therefore LHS = RHS$$

$$9. \quad \cos^2 A + \frac{1}{1 + \cot^2 A} = 1$$

Sol:

$$1 + \cot^2 A = \operatorname{cosec}^2 A \quad \left[\because \operatorname{cosec}^2 A - \cot^2 A = 1 \right]$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\Rightarrow \cot^2 A + \frac{1}{\operatorname{cosec}^2 A}$$

$$\Rightarrow \cos^2 A + \sin^2 A = 1 \quad \therefore LHS = RHS$$

$$10. \quad \sin^2 A + \frac{1}{1 + \tan^2 A} = 1$$

Sol:

$$1 + \tan^2 A = \sec^2 A \quad \left[\because \sec^2 A - \tan^2 A = 1 \right]$$

$$\Rightarrow \sin^2 A + \frac{1}{\sec^2 A} \quad \left[1 + \tan^2 A - \sec^2 A \right]$$

$$\Rightarrow \sin^2 A + \cos^2 A = 1$$

$$\therefore LHS = RHS$$

$$11. \quad \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \operatorname{cosec} \theta - \cot \theta.$$

Sol:

$$L.H.S = \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \quad \text{Rationalize numerator with } \sqrt{1 - \cos \theta}$$

$$\begin{aligned}
 &\Rightarrow \frac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}} \times \frac{\sqrt{1-\cos\theta}}{1-\cos\theta} \\
 &= \frac{(\sqrt{1-\cos\theta})^2}{\sqrt{(1-\cos\theta)(1+\cos\theta)}} \\
 &= \frac{1-\cos\theta}{\sqrt{1-\cos^2\theta}} = \frac{1-\cos\theta}{\sqrt{\sin^2\theta}} = \frac{1-\cos\theta}{\sin\theta} \\
 &= \operatorname{cosec}\theta - \cot\theta
 \end{aligned}$$

12. $\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

Sol:

$$1 = \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2}$$

$$\cos\theta = \cos 2 \cdot \frac{\theta}{2} = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}$$

$$\sin\theta = \sin 2 \cdot \frac{\theta}{2} = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$LHS = \frac{1-\cos\theta}{\sin\theta} = \frac{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}$$

$$\frac{\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}$$

$$= \frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} = \tan \frac{\theta}{2}$$

$$RHS = \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}$$

$$= \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}} = \tan \frac{\theta}{2}$$

$$\therefore L.H.S = R.H.S$$

13. $\frac{\sin \theta}{1 - \cos \theta} - \operatorname{cosec} \theta + \cot \theta$

Sol:

$$LHS = \frac{\sin \theta}{1 - \cos \theta}$$

Rationalizer both Nr and Or with $1 + \cos \theta$

$$\Rightarrow \frac{\sin \theta}{1 - \cos \theta} \times \frac{1 + \cos \theta}{1 + \cos \theta}$$

$$\Rightarrow \frac{\sin \theta (1 + \cos \theta)}{1 - \cos^2 \theta} \quad \left[\because (a - b)(a + b) = a^2 - b^2 \right]$$

$$\Rightarrow \frac{\sin \theta + \sin \theta \cos \theta}{\sin^2 \theta} \quad \left[\because 1 - \cos^2 \theta = \sin^2 \theta \right]$$

$$\Rightarrow \frac{\sin \theta}{\sin^2 \theta} + \frac{\sin \theta \cos \theta}{\sin^2 \theta}$$

$$\Rightarrow \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} \Rightarrow \operatorname{cosec} \theta + \cot \theta$$

$\therefore LHS = RHS$

14. $\frac{1 - \sin \theta}{1 + \sin \theta} - (\sec \theta - \tan \theta)^2$

Sol:

$$LHS = \frac{1 - \sin \theta}{1 + \sin \theta}$$

Rationalize both Nr and Or with $(1 - \sin \theta)$ multiply

$$\Rightarrow \frac{1 - \sin \theta}{1 + \sin \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta}$$

$$\Rightarrow \frac{(1 - \sin \theta)^2}{\cos^2 \theta} \quad \left[\because (1 - \sin \theta)(1 + \sin \theta) = \cos^2 \theta \right]$$

$$\Rightarrow \left[\frac{1 - \sin \theta}{\cos \theta} \right]^2 \Rightarrow \left[\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \right]^2$$

$$\Rightarrow [\sec \theta - \tan \theta]^2$$

$= LHS = RHS$ Hence proved

15. $(\operatorname{cosec} \theta + \sin \theta)(\operatorname{cosec} \theta - \sin \theta) = \cos^2 \theta = \cos^2 \theta$

Sol:

$$LHS \Rightarrow \operatorname{cosec}^2 \theta - \sin^2 \theta \quad \left[(a + b)(a - b) = a^2 - b^2 \right]$$

$$\Rightarrow 1 + \cot^2 \theta - (1 - \cos^2 \theta) \quad \left[\because \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta \sin^2 \theta = 1 - \cos^2 \theta \right]$$

$$\Rightarrow 1 + \cot^2 \theta - 1 + \cos^2 \theta$$

$$\Rightarrow \cot^2 \theta + \cos^2 \theta$$

$$= LHS = RHS \text{ Hence proved}$$

$$16. \quad \frac{(1 + \cot^2 \theta) \tan \theta}{\sec^2 \theta} = \cot \theta$$

Sol:

$$LHS = \frac{(1 + \cot^2 \theta) \tan \theta}{\sec^2 \theta} \quad \left[\because \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta \right]$$

$$\Rightarrow \frac{\operatorname{cosec}^2 \theta \cdot \tan \theta}{\sec^2 \theta} \Rightarrow \frac{1}{\sin^2 \theta} \cdot \frac{\cos^2 \theta}{1} \cdot \frac{\sin \theta}{\cos \theta}$$

$$\Rightarrow \frac{\cos \theta}{\sin \theta} = \cot \theta$$

$$= LHS = RHS \text{ Hence proved}$$

$$17. \quad (\sec \theta + \cos \theta)(\sec \theta - \cos \theta) = \tan^2 \theta + \sin^2 \theta$$

Sol:

$$LHS = \sec^2 \theta - \cos^2 \theta \quad \left[\because (\sec \theta + \cos \theta)(\sec \theta - \cos \theta) = \sec^2 \theta - \cos^2 \theta \right]$$

$$\Rightarrow 1 + \tan^2 \theta - (1 - \sin^2 \theta) \quad \left[\because \sec^2 \theta = 1 + \tan^2 \theta \text{ and } \cos^2 \theta = 1 - \sin^2 \theta \right]$$

$$\Rightarrow 1 + \tan^2 \theta - 1 + \sin^2 \theta$$

$$\tan^2 \theta + \sin^2 \theta$$

$$= LHS = RHS \text{ Hence proved}$$

$$18. \quad \sec A(1 - \sin A)(\sec A + \tan A) = 1$$

Sol:

$$LHS = \frac{1}{\cos A} = (1 - \sin A) \times \left[\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right] \quad \left[\because \sec A = \frac{1}{\cos A} \text{ and } \tan A = \frac{\sin A}{\cos A} \right]$$

$$\Rightarrow \frac{1}{\cos A} \times (1 - \sin A) \frac{(1 + \sin A)}{\cos A}$$

$$= \frac{\cos^2 A}{\cos^2 A} = 1 \quad \left[\because (1 - \sin A)(1 + \sin A) \cdot \cos^2 A = 1 - \sin^2 A \right]$$

$$= LHS = RHS \text{ Hence proved}$$

19. $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$

Sol:

$$LHS = \left[\frac{1}{\sin A} - \sin A \right] \left[\frac{1}{\cos A} - \cos A \right] \left[\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right]$$

$$\Rightarrow \frac{1 - \sin^2 A}{\sin A} \times \frac{1 - \cos^2 A}{\cos A} \times \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$

$$\Rightarrow \frac{\cos^2 A \cdot \sin^2 A \cdot 1}{\sin^2 A \cos^2 A} \quad \left[\begin{array}{l} \because \operatorname{cosec} A = \frac{1}{\sin A} \\ \sec A = \frac{1}{\cos A} \\ \tan A = \frac{\sin A}{\cos A} \\ \cot A = \frac{\cos A}{\sin A} \end{array} \right]$$

$$= 1 \quad \left[\begin{array}{l} \because 1 - \sin^2 A = \cos^2 A \\ 1 - \cos^2 A = \sin^2 A \\ \sin^2 A + \cos^2 A = 1 \end{array} \right]$$

$= LHS = RHS$ Hence proved

20. $\tan^2 \theta - \sin^2 \theta \quad \tan^2 \theta \sin^2 \theta$

Sol:

$$LHS = \tan^2 \theta - \sin^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta \quad \left[\because \tan^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta} \right]$$

$$\Rightarrow \sin^2 \theta \left[\frac{1}{\cos^2 \theta} - 1 \right]$$

$$\sin^2 \theta \left[\frac{1 - \cos^2 \theta}{\cos^2 \theta} \right]$$

$$\Rightarrow \sin^2 \theta \cdot \frac{\sin^2 \theta}{\cos^2 \theta} = \sin^2 \theta \tan^2 \theta$$

$= LHS = RHS$ Hence proved

21. $(1 + \tan^2 \theta)(1 - \sin \theta) \cdot (1 + \sin \theta) = 1$

Sol:

$$LHS = (1 + \tan^2 \theta)(1 - \sin^2 \theta) \quad \left[\because (a - b)(a + b) = a^2 - b^2 \right]$$

$$\Rightarrow \sec^2 \theta \cdot \cos^2 \theta \quad \left[\because \sec^2 \theta = 1 + \tan^2 \theta \right]$$

$$= 1$$

$= LHS = RHS$ Hence proved

22. $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A = 1$

Sol:

$$\begin{aligned} LHS &= \sin^2 A \cdot \frac{\cos^2 A}{\sin^2 A} + \cos^2 A \cdot \frac{\sin^2 A}{\cos^2 A} \\ &= \cos^2 A + \sin^2 A \quad \left[\because \cot^2 A = \cos^2 \frac{A}{\sin^2 A} \tan^2 A = \frac{\sin^2 A}{\cos^2 A} \right] \\ &= LHS = RHS \text{ Hence proved} \end{aligned}$$

23. (i) $\cos \theta - \tan \theta = \frac{2 \cos^2 \theta - 1}{\sin \theta \cos \theta}$

Sol:

$$\begin{aligned} L.H.S &= \frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} \\ &= \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta} = \frac{\cos^2 \theta}{\sin \theta \cos \theta} \quad \left[\because \cos^2 \theta - \sin^2 \theta = \cos \theta \right] \\ &\quad \left[\because \cos^2 = 2 \cos^2 \theta - 1 \right] \end{aligned}$$

$$= \frac{2 \cos^2 \theta - 1}{\sin \theta \cos \theta}$$

$= LHS = RHS$ Hence proved

(ii) $\tan \theta - \cot \theta = \frac{2 \sin^2 \theta - 1}{\sin \theta \cos \theta}$

Sol:

$$\begin{aligned} LHS &= \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta} \\ &\Rightarrow \frac{\sin^2 \theta - \cos^2 \theta}{\cos \theta \sin \theta} \\ &\Rightarrow \frac{\sin^2 \theta - (1 - \sin^2 \theta)}{\cos \theta \sin \theta} \quad \left[\because \cos^2 \theta = 1 - \sin^2 \theta \right] \end{aligned}$$

$$\Rightarrow \frac{2 \sin^2 \theta - 1}{\sin \theta \cos \theta}$$

$\therefore LHS = RHS$ Hence proved



24. $\frac{\cos^2 \theta}{\sin \theta} - \cos \theta \csc \theta + \sin \theta = \theta$

Sol:

$$LHS = \frac{(15^2) - \sin \theta \cos \theta \csc \theta + \sin^2 \theta}{\sin \theta}$$

$$\Rightarrow \frac{\cos^2 \theta + \sin^2 \theta - 1}{\sin \theta} \quad [\because \sin \theta \csc \theta = 1]$$

$$= 0$$

$\therefore LHS = RHS$ Hence proved

25. $\frac{1}{1 + \sin A} + \frac{1}{1 - \sin A} = 2 \sec^2 A$

Sol:

$$LHS = \frac{1 - \sin A + 1 + \sin A}{(1 + \sin A)(1 - \sin A)}$$

$$\Rightarrow \frac{2}{1 - \sin^2 A} \quad [\because (1 + \sin A)(1 - \sin A) = 1 - \sin^2 A]$$

$$\Rightarrow \frac{2}{\cos^2 A} \Rightarrow 2 \sec^2 A \quad [\because 1 - \sin A = \cos A]$$

$\therefore LHS = RHS$ Hence proved

26. $\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos A}{1 + \sin \theta} = 2 \sec \theta$

Sol:

$$LHS = \frac{(1 + \sin \theta)^2 + \cos^2 \theta}{\cos \theta (1 + \sin \theta)}$$

$$= \frac{1 + \sin^2 \theta + 2 \sin \theta + \cos^2 \theta}{\cos \theta (1 + \sin \theta)}$$

$$\Rightarrow \frac{2(1 + \sin \theta)}{\cos \theta (1 + \sin \theta)} = 2 \sec \theta$$

$\therefore LHS = RHS$ Hence proved

27. $\frac{(1 + \sin \theta)^2 + (1 - \sin \theta)^2}{2 \cos^2 \theta} = \frac{1 + \sin^2 \theta}{1 - \sin^2 \theta}$

Sol:

$$LHS = \frac{1 + \sin^2 \theta + 2 \sin \theta + 1 + \sin^2 \theta - 2 \sin \theta}{2 \cos \theta}$$

$$\Rightarrow \frac{2(1+\sin^2 \theta)}{2\cos^2 \theta} \Rightarrow \frac{1+\sin^2 \theta}{1-\sin^2 \theta} \quad \left[\because \cos^2 \theta = 1 - \sin^2 \theta \right]$$

$\therefore LHS = RHS$ Hence proved

28. $\frac{1+\tan^2 \theta}{1+\cot^2 \theta} - \left[\frac{1-\tan \theta}{\cot \theta} \right]^2 - \tan^2 \theta$

Sol:

$$LHS \Rightarrow \frac{1+\tan^2 \theta}{1+\cot^2 \theta} = \frac{\sec^2 \theta}{\operatorname{cosec}^2 \theta} \quad \left[\because \tan^2 \theta + 1 = \sec^2 \theta \quad 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta \right]$$

$$= \frac{1}{\cos^2 \theta \cdot 1} \sin^2 \theta = \tan^2 \theta$$

$$\Rightarrow \left[\frac{1-\tan \theta}{1-\cot \theta} \right]^2 \Rightarrow \left[\frac{1-\tan \theta}{1-\frac{1}{\tan \theta}} \right]^2$$

$$\Rightarrow \left[\frac{1-\tan \theta}{(1-\tan \theta)} \cdot \tan \theta \right]^2 = \tan^2 \theta$$

$\therefore LHS = RHS$ Hence proved

29. $\frac{1+\sec \theta}{\sec \theta} = \frac{\sin^2 \theta}{1-\cos \theta}$

Sol:

$$LHS = \frac{1+\sec \theta}{\sec \theta} = \frac{1+\frac{1}{\cos \theta}}{\frac{1}{\cos \theta}}$$

$$= \frac{\cos \theta + 1}{\cos \theta} \cdot \cos \theta$$

$$= 1 + \cos \theta$$

$$RHS = \frac{\sin^2 \theta}{1-\cos \theta} \Rightarrow \frac{1-\cos^2 \theta}{1-\cos \theta}$$

$$\Rightarrow \frac{(1-\cos \theta)(1+\cos \theta)}{1-\cos \theta} = 1 + \cos \theta$$

$\therefore LHS = RHS$ Hence proved

30. $\frac{\tan \theta}{1 - \cot \theta} = \frac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta.$

Sol:

$$LHS = \frac{\tan \theta}{1 - \frac{1}{\tan \theta}} + \frac{\cot \theta}{1 - \tan \theta}$$

$$\Rightarrow -\frac{\tan^2 \theta}{(1 - \tan \theta)} + \frac{\cot \theta}{1 - \tan \theta}$$

$$\frac{1}{1 - \tan \theta} \left[\frac{1}{\tan \theta} - \tan^2 \theta \right]$$

$$\frac{1}{1 - \tan \theta} \left[\frac{1 - \tan^3 \theta}{\tan \theta} \right]$$

$$\Rightarrow \frac{1}{1 - \tan \theta} \frac{(1 - \tan \theta)(1 + \tan \theta + \tan^2 \theta)}{\tan \theta}$$

$$\left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2) \right]$$

$$\Rightarrow \frac{1 + \tan \theta + \tan^2 \theta}{\tan \theta}$$

$$\Rightarrow \cot \theta + 1 + \tan \theta$$

$\therefore LHS = RHS$ Hence proved

31. $\sec^6 \theta = \tan^6 \theta + 3 \tan^2 \theta \sec^2 \theta + 1$

Sol:

We know that $\sec^2 \theta - \tan^2 \theta = 1$

Cubing on both sides

$$(\sec^2 \theta - \tan^2 \theta)^3 = 1$$

$$\sec^6 \theta - \tan^6 \theta - 3 \sec^2 \theta \tan^2 \theta (\sec^2 \theta - \tan^2 \theta) = 1$$

$$\tan^2 \theta \left[\because (a - b)^3 = a^3 - b^3 - 3ab(a - b) \right]$$

$$\Rightarrow \sec^6 \theta - \tan^6 \theta = 3 \sec^2 \theta \tan^2 \theta + 1$$

$$\Rightarrow \sec^6 \theta = \tan^6 \theta + 1 + 3 \tan^2 \theta \sec^2 \theta$$

Hence proved

32. $\operatorname{cosec}^6 \theta = \cot^6 \theta + 3 \cot^2 \theta \operatorname{cosec}^2 \theta + 1$

Sol:

We know that $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$

Cubing on both sides

$$(\operatorname{cosec}^2 \theta - \cot^2 \theta)^3 = (1)^3$$

$$\Rightarrow \sec^6 \theta - \cot^6 \theta - 3 \sec^2 \theta \cot^2 \theta (\sec^2 \theta - \cot^2 \theta) = 1$$

$$\left[\because (a-b)^3 - a^3 - b^3 - 3ab(a-b) \right]$$

$$\Rightarrow \sec^6 \theta = 1 + 3 \sec^2 \theta \cot^2 \theta + \cot^6 \theta$$

Hence proved

33. $\frac{(1 + \tan^2 \theta) \cot \theta}{\sec^2 \theta} = \tan \theta$

Sol:

$$\sec^2 \theta = \tan^2 \theta + 1$$

$$\therefore \sec^2 \theta = 1 + \tan^2 \theta$$

$$LHS = \frac{\sec^2 \theta \cdot \cot \theta}{\sec^2 \theta} \Rightarrow \frac{1 \cdot \sin^6 \theta \cdot \cos \theta}{\cos^2 \theta \cdot \sin \theta}$$

$$\left[\because \sec \theta = \frac{1}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta} \right]$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = \tan \theta$$

$\therefore LHS = RHS$ Hence proved

34. $\frac{1 + \cos A}{\sin^2 A} = \frac{1}{1 - \cos A}$

Sol:

We know that $\sin^2 A + \cos^2 A = 1$

$$\sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = (1 - \cos A)(1 + \cos A)$$

$$\Rightarrow LHS = \frac{(1 + \cos A)}{(1 - \cos A)(1 + \cos A)} = \frac{1}{1 - \cos A}$$

$\therefore L.H.S = R.H.S$ Hence proved

35. $\frac{\sec A - \tan A}{\sec A + \tan A} = \frac{\cos^2 A}{(1 + \sin A)^2}$

Sol:

$$LHS = \frac{\sec \theta - \tan \theta}{\sec A + \tan A}$$

Rationalizing the denominator by multiply and dividing with $\sec A + \tan A$ we get

$$\frac{(\sec A - \tan A)}{(\sec A + \tan A)} \times \frac{(\sec A + \tan A)}{(\sec A + \tan A)} = \frac{\sec^2 A - \tan^2 A}{(\sec A + \tan A)^2} = \frac{1}{(\sec A + \tan A)^2}$$

$$[\because \sec^2 A - \tan^2 A = 1]$$

$$= \frac{1}{\sec^2 A + \tan^2 A + 2\sec A \tan A} = \frac{1}{\frac{1}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A} + \frac{2\sin A}{\cos^2 A}}$$

$$\Rightarrow \frac{\cos^2}{1 + \sin^2 A + 2\sin A} = \frac{\cos^2 A}{(1 + \sin A)^2}$$

$\therefore L.H.S = R.H.S$ Hence proved

36. $\frac{1 + \cos A}{\sin A} = \frac{\sin A}{1 - \cos A}$

Sol:

$$LHS = \frac{1 + \cos A}{\sin A} \quad \dots(1)$$

Multiply both Nr and Dr with $(1 - \cos A)$ we get

$$\frac{(1 + \cos A)(1 - \cos A)}{\sin A(1 - \cos A)} = \frac{1 - \cos^2 A}{\sin A(1 - \cos A)}$$

$$= \frac{\sin^2 A}{\sin A(1 - \cos A)} = \frac{1 - \cos^2 A}{\sin A(1 - \cos A)}$$

$$= \frac{\sin^2}{\sin A(1 - \cos A)} \quad [\because \cos^2 A = \sin^2 A]$$

$$= \frac{\sin A}{1 - \cos A}$$

$\therefore L.H.S = R.H.S$ Hence proved

37. $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sin A + \tan A$

Sol:

$$LHS = \sqrt{\frac{1 + \sin A}{1 - \sin A}}$$

Rationalize the Nr. By multiplying both Nr and Dr with $\sqrt{1 + \sin A}$.

$$\Rightarrow \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 + \sin A)(1 - \sin A)}} = \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} \quad [\because (1 + \sin A)(1 - \sin A) = \cos^2 A]$$

$$= \frac{1 + \sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$\sec A + \tan A$$

$\therefore L.H.S = R.H.S$ Hence proved

$$38. \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \operatorname{cosec} A - \cot A$$

Sol:

Rationalizing both Nr and Or by multiplying both with $\sqrt{1 - \cos A}$ we get

$$\Rightarrow \sqrt{\frac{(1 - \cos A)(1 - \cos A)}{(1 + \cos A)(1 - \cos A)}} \quad \left[\because (1 + \cos A)(1 - \cos A) = 1 - \cos^2 A = \sin^2 A \right]$$

$$\sqrt{\frac{(1 - \cos A)^2}{\sin^2 A}}$$

$$= \frac{1 - \cos A}{\sin A}$$

$$= \operatorname{cosec} A - \cot A$$

$\therefore L.H.S = R.H.S$ Hence proved

$$39. (\sec A - \tan A)^2 = \frac{1 - \sin A}{1 + \sin A}$$

Sol:

$$LHS = (\sec A - \tan A)^2$$

$$\Rightarrow \left[\frac{1}{\cos A} - \frac{\sin A}{\cos A} \right]^2 \Rightarrow \frac{(1 - \sin A)^2}{\cos^2 A}$$

$$\Rightarrow \frac{(1 - \sin A)^2}{1 - \sin^2 A} \quad \left[\because 1 - \sin^2 A = \cos^2 A \right]$$

$$\Rightarrow \frac{(1 - \sin A)^2}{(1 - \sin A)(1 + \sin A)} \quad \left[\because a^2 - b^2 = (a - b)(a + b) \right]$$

$$= \frac{1 - \sin A}{1 + \sin A}$$

$\therefore L.H.S = R.H.S$ Hence proved



40. $\frac{1 - \cos A}{1 + \cos A} = (\cot A - \operatorname{cosec} A)^2$

Sol:

$$LHS = \frac{1 - \cos A}{1 + \cos A}$$

Rationalizing Nr by multiplying and dividing with $1 - \cos A$.

$$= \frac{(1 - \cos A)(1 - \cos A)}{(1 + \cos A)(1 - \cos A)}$$

$$\Rightarrow \frac{(1 - \cos A)^2}{1 - \cos^2 A}$$

$$\Rightarrow \frac{(1 - \cos A)^2}{\sin^2 A} \quad \left[\because (a + b)(a - b) = a^2 - b^2 \quad 1 - \cos^2 A = \sin^2 A \right]$$

$$= \left[\frac{1}{\sin A} - \frac{\cos A}{\sin A} \right]^2 \quad (\operatorname{cosec} A - \cot A)^2$$

$$= (\cot A - \operatorname{cosec} A)^2$$

$\therefore L.H.S = R.H.S$ Hence proved

41. $\frac{1}{\sec A - 1} = \frac{1}{\sec A + 1} = 2 \operatorname{cosec} A \cot A$

Sol:

$$LHS = \frac{\sec A + 1 + \sec A - 1}{(\sec A + 1)(\sec A - 1)} = \frac{2 \sec A}{(\sec^2 A - 1)}$$

$$\left[\because (a + b)(a - b) = a^2 - b^2 \quad \sec^2 A - 1 = \tan^2 A \right]$$

$$\Rightarrow \frac{2 \sec A}{\tan^2 A} = \frac{2 \cdot 1 \cos^2 A}{\cos A \cdot \sin^2 A} \quad \left[\because \sec A = \frac{1}{\cos A} \quad \tan^2 A = \frac{\sin^2 A}{\cos^2 A} \right]$$

$$\Rightarrow 2 \operatorname{cosec} A \cot A$$

$\therefore L.H.S = R.H.S$ Hence proved

42. $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{(1 - \cot A)} = \sin A + \cos A$

Sol:

$$LHS = \frac{\cos A}{1 - \tan A} + \frac{\sin A}{\left(1 - \frac{1}{\tan A}\right)}$$

$$= \frac{\cos A}{1 - \tan A} - \frac{\sin A \cdot \tan A}{1 - \tan A}$$

$$\begin{aligned}
&\Rightarrow \frac{\cos A - \sin A \tan A}{(1 - \tan A)} \\
&\Rightarrow \frac{\cos A - \sin A \cdot \frac{\sin A}{\cos A}}{1 - \frac{\sin A}{\cos A}} \\
&\Rightarrow \frac{\cos^2 A - \sin^2 A \cos A}{(\cos A - \sin A) \cos A} = \frac{(\cos A - \sin A)(\cos A + \sin A)}{\cos A - \sin A} \\
&\Rightarrow \cos A + \sin A. \\
&\therefore L.H.S = R.H.S \text{ Hence proved}
\end{aligned}$$

43. $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2 \sec^2 A$

Sol:

$$\begin{aligned}
&LHS \operatorname{cosec} A \left[\frac{\operatorname{cosec} A + 1 + \operatorname{cosec} A - 1}{\operatorname{cosec}^2 A - 1} \right] \quad \left[\because \operatorname{cosec}^2 A - 1 = \cot^2 A \right] \\
&\Rightarrow \operatorname{cosec} A \left[\frac{2 \operatorname{cosec} A}{\cot^2 A} \right] \\
&\Rightarrow \frac{2}{\sin^2 A} \frac{\sin^2 A}{\cos^2 A} = 2 \sec^2 A. \\
&\therefore LHS = RHS \text{ Hence proved.}
\end{aligned}$$

44. $(1 + \tan^2 A) + \left(1 + \frac{1}{\tan^2 A} \right) = \frac{1}{\sin^2 A - \sin^4 A}$

Sol:

$$\begin{aligned}
&LHS = \left[1 + \frac{\sin^2 A}{\cos^2 A} \right] + \left[1 + \frac{\cos^2 A}{\sin^2 A} \right] \\
&\Rightarrow \frac{\cos^2 A + \sin^2 A}{\cos^2 A} + \frac{\sin^2 A + \cos^2 A}{\sin^2 A} \\
&\Rightarrow \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} \quad \left[\because \sin^2 A + \cos^2 A = 1 \right] \\
&\Rightarrow \frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A} = \frac{1}{\sin^2 A (1 - \sin^2 A)} \quad \left[\cos^2 A = 1 - \sin^2 A \right] \\
&\Rightarrow \frac{1}{\sin^2 A - \sin^4 A} \\
&\therefore LHS = RHS \text{ Hence proved.}
\end{aligned}$$

45. $\frac{\tan^2 A}{1 + \tan^2 A} + \frac{\cot^2 A}{1 + \cot A}$

Sol:

We know that

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\therefore LHS = \frac{\tan^2 A}{\sec^2 A} + \frac{\cot^2 A}{\operatorname{cosec}^2 A}$$

$$\Rightarrow \frac{\sin^2 A}{\cos^2 A} \times \frac{\cos^2 A}{1} + \frac{\cos^2 A}{\sin^2 A} = \frac{\sin^2 A}{1}$$

$$\left[\because \tan A = \frac{\sin A}{\cos A} \sec A = \frac{1}{\cos A} \cot A = \frac{\cos A}{\sin A} \operatorname{cosec} A = \frac{1}{\sin A} \right]$$

$$\Rightarrow \sin^2 A + \cos^2 A$$

$$= 1$$

$\therefore LHS = RHS$ Hence proved.

46. $\frac{\cot A - \cos A}{\cos A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

Sol:

$$= \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A} \quad \left[\because \cot A = \frac{\cos A}{\sin A} \right]$$

$$= \frac{\cos A \left[\frac{1}{\sin A} - 1 \right]}{\cos A \left[\frac{1}{\sin A} + 1 \right]}$$

$$= \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$$

47. (i) $\frac{1 + \cos \theta + \sin \theta}{1 + \cos \theta - \sin \theta}$

(ii) $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1}$

(iii) $\frac{\cos \theta - \sin \theta + 1}{\cos \theta + \sin \theta - 1} = \operatorname{cosec} \theta + \cot \theta$

Sol:

$$(i) \Rightarrow \frac{1 + \cos \theta + \sin \theta}{1 + \cos \theta - \sin \theta}$$

Dividing the equation with $\cos \theta$ we get or both Nr and Dr

$$\begin{aligned} \frac{1 + \cos \theta + \sin \theta}{\cos \theta} &= \frac{1}{\cos \theta} + \frac{\cos \theta}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \\ \frac{1 + \cos \theta - \sin \theta}{\cos \theta} &= \frac{1}{\cos \theta} - \frac{\cos \theta}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \\ &= \frac{\sec \theta + 1 + \tan \theta}{\sec \theta + 1 - \tan \theta} \\ &= \frac{\sec \theta + \tan \theta + \sec^2 \theta - \tan^2 \theta}{\sec^2 \theta - \tan^2 \theta + 1} \quad \left[\because \sec^2 \theta - \tan^2 \theta = 1 \right] \end{aligned}$$

Or

$$\begin{aligned} &\frac{\sec \theta + \tan \theta + 1}{\sec \theta - \tan \theta + 1} \\ &\frac{\frac{1}{\sec \theta - \tan \theta} + 1}{\sec \theta - \tan \theta + 1} \quad \left[\because \sec \theta + \tan \theta = \frac{1}{\sec \theta - \tan \theta} \right] \end{aligned}$$

Or

$$\begin{aligned} &\frac{\sec \theta + \tan \theta + 1}{\sec \theta - \tan \theta + 1} \\ &\frac{\frac{1}{\sec \theta - \tan \theta} + 1}{\sec \theta - \tan \theta + 1} \quad \left[\because \sec \theta + \tan \theta = \frac{1}{\sec \theta - \tan \theta} \right] \\ &= \frac{1 + \sec \theta - \tan \theta}{1 + \sec \theta - \tan \theta} \times \frac{1}{\sec \theta - \tan \theta} \\ &= \frac{1}{\sec \theta - \tan \theta} = \sec \theta + \tan \theta \\ &= \sec \theta + \tan \theta \\ &= \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \\ &= \frac{1 + \sin \theta}{\cos \theta} \end{aligned}$$

$$(ii) \frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1}$$

Divide Nr and Dr with $\cos \theta$, we get

$$\begin{aligned}
 & \frac{\frac{\sin \theta - \cos \theta + 1}{\cos \theta}}{\frac{\sin \theta + \cos \theta - 1}{\cos \theta}} = \frac{\tan \theta - 1 + \sec \theta}{\tan \theta + 1 - \sec \theta} \\
 & = \frac{1}{\sec \theta - \tan \theta} - 1 \\
 & = \frac{1 - \sec \theta + \tan \theta}{1 - \sec \theta + \tan \theta} \times \frac{1}{\sec \theta - \tan \theta} \\
 & = \frac{1}{\sec \theta - \tan \theta}
 \end{aligned}$$

$$(iii) \frac{\cos \theta - \sin \theta + 1}{\cos \theta + \sin \theta - 1} = \operatorname{cosec} \theta + \cot \theta$$

Divide both Nr and Dr with $\sin \theta$

$$\begin{aligned}
 & \frac{\frac{\cos \theta - \sin \theta + 1}{\sin \theta}}{\frac{\cos \theta + \sin \theta - 1}{\sin \theta}} \\
 & = \frac{\cot \theta - 1 + \operatorname{cosec} \theta}{\cot \theta + 1 - \operatorname{cosec} \theta} \\
 & = \frac{\cot \theta + \operatorname{cosec} \theta - (\operatorname{cosec}^2 \theta - \cot^2 \theta)}{\cot \theta - \operatorname{cosec} \theta + 1} \\
 & = \frac{\cot \theta + \operatorname{cosec} \theta - (\operatorname{cosec}^2 \theta + \cot^2 \theta)}{\cot \theta - \operatorname{cosec} \theta + 1} \\
 & = \frac{\cot \theta + \operatorname{cosec} (1 - (\operatorname{cosec} - \cot \theta))}{\cot \theta - \operatorname{cosec} \theta + 1} \\
 & = \cot \theta + \operatorname{cosec} \theta
 \end{aligned}$$

$$48. \frac{1}{\sec A + \tan A} - \frac{1}{\cos A} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$$

Sol:

$$\begin{aligned}
 LHS : \sec A - \tan A & \left[\because \frac{1}{\sec A + \tan A} = \sec A - \tan A \right] \\
 & = -\tan A
 \end{aligned}$$

$$\begin{aligned}
 &RHS \frac{1}{\cos A} - \frac{1}{\sec A - \tan A} \\
 &\sec A - (\sec A + \tan A) \\
 &\left[\because \frac{1}{\sec A - \tan A} = \sec A + \tan A \right] \\
 &= -\tan A \\
 &LHS = RHS
 \end{aligned}$$

49. $\tan^2 A + \cot^2 A = \sec^2 A \operatorname{cosec}^2 A - 2$

Sol:

$$\begin{aligned}
 \tan^2 A + \cot^2 A &= \frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\sin^2 A} \\
 &= \frac{\sin^4 A + \cos^4 A}{\cos^2 A \sin^2 A} \\
 &= \frac{1 - 2\sin^2 A \cos^2 A}{\sin^2 A \cos^2 A} \quad \left[\because \sin^4 A + \cos^4 A = 1 - 2\sin^2 A \cos^2 A \right] \\
 &= \sec^2 A \operatorname{cosec}^2 A - 2 \\
 \sin^4 A + \cos^4 A &\text{ is in the form of } a^4 + b^4 \\
 a^4 + b^4 &= (a^2 + b^2)^2 - 2a^2 b^2 \\
 \text{Here } a &= \sin A, b = \cos A \\
 &= (\sin^2 A + \cos^2 A)^2 - 2\sin^2 A \cos^2 A \\
 &= 1 - 2\sin^2 A \cos^2 A
 \end{aligned}$$

50. $\frac{1 - \tan^2 A}{\cot^2 A - 1} = \tan^2 A.$

Sol:

$$\begin{aligned}
 1 - \frac{\sin^2 A}{\cos^2 A} &= \frac{\cos^2 A - \sin^2 A}{\cos^2 A} \\
 \frac{\cos^2 A}{\sin^2 A} - 1 &= \frac{\cos^2 A - \sin^2 A}{\sin^2 A} \\
 &= \frac{\sin^2 A}{\cos^2 A} \\
 &= \tan^2 A.
 \end{aligned}$$



51. $1 + \frac{\cot^2 \theta}{1 + \operatorname{cosec} \theta} = \operatorname{cosec} \theta$

Sol:

$$1 + \frac{\operatorname{cosec}^2 \theta - 1}{1 + \operatorname{cosec} \theta} \quad \left[\because \operatorname{cosec}^2 \theta - \cot^2 \theta = 1, \cot^2 \theta = \operatorname{cosec}^2 \theta - 1 \right]$$

$$1 + \frac{(\operatorname{cosec} \theta - 1)(\operatorname{cosec} \theta + 1)}{1 + \operatorname{cosec} \theta}$$

$$= 1 + \operatorname{cosec} \theta - 1 \quad \left[\because (a+b)(a-b) = a^2 - b^2, a = \operatorname{cosec} \theta, b = 1. \right]$$

$$= \operatorname{cosec} \theta$$

52. $\frac{\cos \theta}{\operatorname{cosec} \theta + 1} + \frac{\cos \theta}{\operatorname{cosec} \theta - 1} = 2 \tan \theta$

Sol:

$$\frac{\cos \theta}{\frac{1}{\sin \theta} + 1} + \frac{\cos \theta}{\frac{1}{\sin \theta} - 1}$$

$$\frac{\cos \theta}{1 + \sin \theta} + \frac{\cos \theta}{1 - \sin \theta}$$

$$\frac{(\cos \theta)(\sin \theta)}{1 + \sin \theta} + \frac{(\cos \theta)(\sin \theta)}{1 - \sin \theta}$$

$$\frac{(1 - \sin \theta)(\sin \theta \cos \theta) + (1 + \sin \theta)(\sin \theta \cos \theta)}{(1 + \sin \theta)(1 - \sin \theta)}$$

$$\frac{\sin \theta \cos \theta - \sin^2 \theta \cos \theta + \sin \theta \cos \theta + \sin^2 \theta \cos \theta}{1 - \sin^2 \theta}$$

$$= \frac{2 \sin \theta \cos \theta}{\cos^2 \theta}$$

$$= \frac{2 \sin \theta}{\cos \theta}$$

$$= 2 \tan \theta$$

53. $\frac{1 + \cos \theta - \sin^2 \theta}{\sin \theta (1 + \cos \theta)} = \cot \theta$

Sol:

$$\begin{aligned} & \frac{1 + \cos \theta - \sin^2 \theta}{\sin \theta (1 + \cos \theta)} \\ &= \frac{1 - \sin^2 \theta + \cos \theta}{\sin \theta (1 + \cos \theta)} \\ &= \frac{\cos^2 \theta + \cos \theta}{\sin \theta (1 + \cos \theta)} \\ &= \frac{\cos \theta (1 + \cos \theta)}{\sin \theta (1 + \cos \theta)} \\ &= \cot \theta. \end{aligned}$$

54. $\frac{\tan^3 \theta}{1 + \tan^2 \theta} + \frac{\cot^3 \theta}{1 + \cot^2 \theta} = \sec \theta \operatorname{cosec} \theta - 2 \sin \theta \cos \theta$

Sol:

$$\frac{\tan^3 \theta}{\sec^2 \theta} + \frac{\cos^2 \theta}{\operatorname{cosec}^2 \theta} \quad \left[\because \sec^2 \theta - \tan^2 \theta = 1, \operatorname{cosec}^2 \theta - \cot^2 \theta = 1 \right]$$

$$\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta.$$

$$\tan \theta + \cos^2 \theta + \cot^3 \theta \times \sin^3 \theta \quad \left[\because \frac{1}{\sec^2 \theta} = \cos^2 \theta, \frac{1}{\operatorname{cosec}^2 \theta} = 1 + \cot^2 \theta \right]$$

$$\frac{\sin^3 \theta}{\cos^3 \theta} \times \cos^2 \theta + \frac{\cos^3 \theta}{\sin^3 \theta} \times \sin^2 \theta$$

$$\frac{\sin^3 \theta}{\cos \theta} + \frac{\cos^3 \theta}{\sin \theta}$$

$$= \frac{\sin^4 \theta + \cos^4 \theta}{\sin \theta \cos \theta}$$

$$\frac{1 - 2 \sin^2 \theta \cos^2 \theta}{\sin \theta \cos \theta}$$

$$\frac{1}{\sin \theta \cos \theta} - \frac{2 \sin^2 \theta \cos^2 \theta}{\sin \theta \cos \theta}$$

$$\sec \theta \operatorname{cosec} \theta - 2 \sin \theta \cos \theta.$$

55. If $T_n = \sin^n \theta + \cos^n \theta$, prove that $\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$.

Sol:

$$\begin{aligned}
 LHS &= \frac{(\sin^3 \theta + \cos^3 \theta) - (\sin^5 \theta + \cos^5 \theta)}{\sin \theta + \cos \theta} \\
 &= \frac{\sin^3 \theta (1 - \sin^2 \theta) + \cos^3 \theta (1 - \cos^2 \theta)}{\sin \theta + \cos \theta} \\
 &= \frac{\sin^3 \theta \times \cos^2 \theta + \cos^3 \theta \times \sin^2 \theta}{\sin \theta + \cos \theta} \\
 &= \frac{\sin^2 \theta + \cos \theta (\sin \theta + \cos \theta)}{\sin \theta + \cos \theta} \\
 &= \sin^3 \theta \cos^2 \theta \\
 \frac{T_5 - T_7}{T_3} &= \frac{(\sin^5 \theta + \cos^5 \theta) - (\sin^7 \theta + \cos^7 \theta)}{\sin^3 \theta + \cos^3 \theta} \\
 &= \frac{\sin^5 \theta (1 - \sin^2 \theta) + \cos^5 \theta (1 - \cos^2 \theta)}{\sin^3 \theta + \cos^3 \theta} \\
 &= \frac{\sin^5 \theta + \cos^2 \theta + \cos^5 \theta (\sin^2 \theta)}{\sin^3 \theta + \cos^3 \theta} \\
 &= \frac{\sin^2 \theta \cos^2 \theta (\sin^3 \theta + \cos^3 \theta)}{\sin^3 \theta + \cos^3 \theta} \\
 &= \sin^2 \theta \cos^2 \theta \\
 L.H.S &= R.H.S \text{ Hence Proval.} \\
 &= \frac{\sin^2 \theta \cos^2 \theta (\sin \theta + \cos \theta)}{\sin^2 \theta + \cos \theta} \\
 &= \sin^2 \theta \cos^2 \theta \\
 L.H.S &= R.H.S
 \end{aligned}$$

56. $\left[\tan \theta + \frac{1}{\cos \theta} \right]^2 + \left[\tan \theta - \frac{1}{\cos \theta} \right]^2 = 2 \left(\frac{1 + \sin^2 \theta}{1 - \sin^2 \theta} \right)$

Sol:

$$\begin{aligned}
 &\Rightarrow (\tan \theta + \sec \theta)^2 + (\tan \theta - \sec \theta)^2 \\
 &= \tan^2 \theta + \sec^2 \theta + 2 \tan \theta \sec \theta + \tan^2 \theta + \sec^2 \theta + 2 \tan \theta \sec \theta.
 \end{aligned}$$

$$\begin{aligned}
&= 2 \tan^2 \theta + 2 \sec^2 \theta \\
&= 2 \left[\tan^2 \theta + \sec^2 \theta \right] \\
&= 2 \left[\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{1}{\cos^2 \theta} \right] \\
&= 2 \left(\frac{\sin + \sin^2 \theta}{\cos^2 \theta} \right)
\end{aligned}$$

$$57. \left[\frac{1}{\sec^2 \theta - \cos^2 \theta} + \frac{1}{\operatorname{cosec}^2 \theta - \sin^2 \theta} \right] \sin^2 \theta \cos^2 \theta = \frac{1 - \sin^2 \theta \cos^2 \theta}{2 + \sin^2 \theta \cos^2 \theta}.$$

Sol:

$$\begin{aligned}
&\Rightarrow \left[\frac{1}{\frac{1}{\cos^2 \theta} - \cos^2 \theta} + \frac{1}{\operatorname{cosec}^2 \theta - \sin^2 \theta} \right] \sin^2 \theta \cos^2 \theta. \\
&= \left[\frac{1}{\frac{1 - \cos^4 \theta}{\cos^2 \theta}} + \frac{1}{\frac{1 - \sin^4 \theta}{\sin^2 \theta}} \right] \sin^2 \theta \cos^2 \theta. \\
&= \left[\frac{\cos^2 \theta}{1 - \cos^4 \theta} + \frac{\sin^2 \theta}{1 - \sin^4 \theta} \right] \sin^2 \theta \cos^2 \theta \\
&= \left[\frac{\cos^2 \theta}{\cos^2 \theta + \sin^2 \theta - \cos^4 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta + \sin^2 \theta - \sin^4 \theta} \right] \sin^2 \theta \cos^2 \theta. \\
&= \left[\frac{\cos^2 \theta}{\cos^2 \theta (1 - \cos^2 \theta) + \sin^2 \theta} + \frac{\sin^2 \theta}{\sin^2 \theta (1 - \sin^2 \theta) + \cos^2 \theta} \right] \sin^2 \theta \cos^2 \theta. \\
&= \left[\frac{\cos^2 \theta}{\cos^2 \theta \sin^2 \theta + \sin^2 \theta} + \frac{\sin^2 \theta}{\sin^2 \theta \cos^2 \theta + \cos^2 \theta} \right] \sin^2 \theta \cos^2 \theta \\
&= \left[\frac{\cos^2 \theta}{\sin^2 \theta (\cos^2 \theta + 1)} + \frac{\sin^2 \theta}{\cos^2 \theta (\sin^2 \theta + 1)} \right] \sin^2 \theta \cos^2 \theta. \\
&= \left[\frac{\cos^4 \theta (1 + \sin^2 \theta) + \sin^4 \theta (1 + \cos^2 \theta)}{\sin^2 \theta \cos^2 \theta (1 + \cos^2 \theta) (1 + \sin^2 \theta)} \right] \sin^2 \theta \cos^2 \theta \\
&= \frac{\cos^4 \theta (1 + \sin^2 \theta) + \sin^4 \theta (1 + \cos^2 \theta)}{(1 + \cos^2 \theta) (1 + \sin^2 \theta)}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\cos^4 \theta + \cos^4 \theta \sin^2 \theta + \sin^4 \theta + \sin^4 \theta \cos^2 \theta}{1 + \sin^2 \theta + \cos^2 \theta + \cos^2 \theta \sin^2 \theta} \\
&= \frac{1 - 2\sin^2 \theta \cos^2 \theta + \sin^2 \theta \cos^2 \theta (\cos^2 \theta + \sin^2 \theta)}{1 + 1 + \cos^2 \theta \sin^2 \theta} \quad (\because \cos^2 \theta + \sin^2 \theta = 1) \\
&= \frac{1 - \sin^2 \theta \cos^2 \theta}{2 + \sin^2 \theta \cos^2 \theta}
\end{aligned}$$

58. $\left[\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} \right]^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$

Sol:

$$\begin{aligned}
&\Rightarrow \left(\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} \times \frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta - \cos \theta} \right)^2 \\
&\Rightarrow \left[\frac{(1 + \sin \theta - \cos \theta)^2}{(1 + \sin \theta)^2 - \cos^2 \theta} \right] \\
&= \left[\frac{(1)^2 + \sin^2 \theta + \cos^2 \theta + 2 \times 1 \times \sin \theta + 2 \times \sin \theta (-\cos \theta) - 2 \cos \theta}{1 - \cos^2 \theta + \sin^2 \theta + 2 \sin \theta} \right]
\end{aligned}$$

(Since, $\sin^2 \theta + \cos^2 \theta = 1$)

$$\begin{aligned}
&= \left[\frac{1 + 1 + 2 \sin \theta - 2 \sin \theta \cos \theta - 2 \cos \theta}{\sin^2 \theta - \sin^2 \theta + 2 \sin \theta} \right]^2 \\
&= \left[\frac{2 \times 2 \sin \theta - 2 \sin \theta \cos \theta - 2 \cos \theta}{2 \sin^2 \theta + 2 \sin \theta} \right]^2 \\
&= \left[\frac{2(1 + \sin \theta) - 2 \cos \theta(\sin \theta + 1)}{2 \sin \theta(\sin \theta + 1)} \right]^2 \\
&= \left[\frac{(1 + \sin \theta)(2 - 2 \cos \theta)}{2 \sin \theta(\sin \theta + 1)} \right]^2 \\
&= \left[\frac{2 - 2 \cos \theta}{2 \sin \theta} \right]^2
\end{aligned}$$

$$= \left[\frac{2}{2} - \left(\frac{1 - \cos \theta}{\sin \theta} \right) \right]^2$$

$$\begin{aligned}
 &= \left[\frac{1 - \cos \theta}{\sin \theta} \right]^2 \\
 &= \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta} \\
 &= \frac{(1 - \cos \theta) \times (1 - \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)} \\
 &= \frac{1 - \cos \theta}{1 + \cos \theta}.
 \end{aligned}$$

59. $(\sec A + \tan A - 1)(\sec A - \tan A + 1) = 2 \tan A$

Sol:

$$\begin{aligned}
 &= (\sec A + \tan A - \{\sec^2 A - \tan^2 A\}) [\sec A - \tan A + (\sec^2 A - \tan^2 A)] \\
 &= (\sec A + \tan A - \sec A + \tan A)(\sec A - \tan A)(\sec A - \tan A + (\sec A + \tan A)(\sec A - \tan A)) \\
 &= (\sec A + \tan A)(1 - (\sec A - \tan A))(\sec A - \tan A)(1 + \sec A \tan A) \\
 &= (\sec A + \tan A)(1 - \sec A + \tan A)(\sec A - \tan A)(1 + \sec A \tan A) \\
 &= (\sec A + \tan A)(\sec A - \tan A)(1 - \sec A + \tan A)(1 + \sec A \tan A) \\
 &= (\sec^2 A - \tan^2 A)(1 - \sec A + \tan A)(1 - \sec A \tan A) \\
 &= \left[1 - \frac{1}{\cos A} + \frac{\sin A}{\cos A} \right] \left[1 + \frac{1}{\cos A} + \frac{\sin A}{\cos A} \right] \\
 &= \left(\frac{\cos A - 1 + \sin A}{\cos A} \right) \left(\frac{\cos A + 1 + \sin A}{\cos A} \right) \\
 &= \left(\frac{\cos A + \sin^2 A - 1}{\cos^2 A} \right) \\
 &= \frac{\cos^2 A + \sin^2 A + 2 \sin A \cos A - 1}{\cos^2 A} \\
 &= \frac{1 + 2 \sin A \cos A}{\cos^2 A} - 1 \\
 &= \frac{2 \sin A \cos A}{\cos^2 A} \quad \left[\because \sin^2 A + \cos^2 A = 1 \right] \\
 &= 2 \tan A
 \end{aligned}$$

60. $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

Sol:

$$\begin{aligned}
 LHS &= (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) \\
 &= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right) \left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right) \\
 &= \left(\frac{\sin A + \cos A - 1}{\sin A}\right) \left(\frac{\cos A + \sin A + 1}{\cos A}\right) \\
 &= \frac{(\sin A + \cos A)^2 - 1}{\sin A \cos A} \\
 &= \frac{1 + 2 \sin A \cos A - 1}{\sin A \cos A} \quad \left[\because \sin^2 A + \cos^2 A = 1\right] \\
 &= 2.
 \end{aligned}$$

61. $(\operatorname{cosec} \theta - \sec \theta)(\cot \theta - \tan \theta)(\operatorname{cosec} \theta + \sec \theta)(\sec \theta \operatorname{cosec} \theta - 2)$

Sol:

LHS

$$\begin{aligned}
 &(\operatorname{cosec} \theta - \sec \theta)(\cot \theta - \tan \theta) \\
 &\left[\frac{1}{\sin \theta} - \frac{1}{\cos \theta}\right] \left[\frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta}\right] \\
 &\left[\frac{\cos \theta - \sin \theta}{\sin \theta \cos \theta}\right] \left[\frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta}\right] \\
 &\left[\frac{(\cos \theta - \sin \theta)^2 (\cos \theta + \sin \theta)}{\cos^2 \theta \sin^2 \theta}\right]
 \end{aligned}$$

RHS

$$\begin{aligned}
 &(\operatorname{cosec} \theta + \sec \theta)(\sec \theta \operatorname{cosec} \theta - 2) \\
 &= \left[\frac{1}{\sin \theta} + \frac{1}{\cos \theta}\right] \left[\frac{1}{\cos \theta} - \frac{1}{\sin \theta} - 2\right] \\
 &= \left[\frac{\sin \theta - \cos \theta}{\sin \theta \cos \theta}\right] \left[\frac{1 - 2 \sin \theta \cos \theta}{\sin \theta \cos \theta}\right] \\
 &= \left[\frac{\sin \theta + \cos \theta}{\sin \theta \cos \theta}\right] \left[\frac{\cos^2 \theta + \sin^2 \theta - 2 \sin \theta \cos \theta}{\sin \theta \cos \theta}\right] \\
 &= \frac{(\cos \theta + \sin \theta)(\cos \theta - \sin \theta)^2}{\sin^2 \theta \cos^2 \theta} \quad \left[\because \cos^2 \theta + \sin^2 \theta = 1\right]
 \end{aligned}$$

L.H.S = R.H.S Hence proved

62. $(\sec A - \csc A)(1 + \tan A + \cot A) = \tan A \sec A - \cot A \csc A$

Sol:

$$\begin{aligned}
 LHS &= (\sec A - \csc A)(1 + \tan A + \cot A) \\
 &= \left[\frac{1}{\cos A} - \frac{1}{\sin A} \right] \left[1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right] \\
 &= \left[\frac{\sin A - \cos A}{\sin A \cos A} \right] \left[\frac{\cos A \sin A + \sin^2 A + \cos^2 A}{\sin A \cos A} \right] \\
 &= \frac{(\sin A - \cos A)(\sin^2 A + \cos A \sin A + \cos^2 A)}{\sin^2 A \cos^2 A} \\
 &= \frac{(\sin^3 A - \cos^3 A)}{\sin^2 A \cos^2 A} \quad \left[\because (a-b)(a^2+ab)+b=(a^3-b^3) \right]
 \end{aligned}$$

$$RHS = \tan A \sec A - \cot A \csc A$$

$$\begin{aligned}
 &= \frac{\sin A}{\cos A} \times \frac{1}{\cos A} - \frac{\cos A}{\sin A} \times \frac{1}{\cos A} \\
 &= \frac{\sin A}{\cos^2 A} - \frac{\cos A}{\sin A} \\
 &= \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cos^2 A}
 \end{aligned}$$

$L.H.S = R.H.S$ Hence proved.

63. $\frac{\cos A \csc A - \sin A \sec A}{\cos A + \sin A} = \csc A - \sec A$

Sol:

$$\begin{aligned}
& LHS \frac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A} \\
&= \frac{\cos A \times \frac{1}{\sin A} - \sin A \times \frac{1}{\cos A}}{\cos A + \sin A} \\
&= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\cos A}}{\cos A + \sin A} \\
&= \frac{\frac{\cos^2 A - \sin^2 A}{\sin A \cos A}}{\cos A + \sin A} \\
&= \frac{\cos^2 A - \sin^2 A}{\sin A \cos A} \times \frac{1}{\cos A + \sin A} \\
&= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\sin A \cos A (\cos A + \sin A)} \\
&= \frac{\cos A - \sin A}{\sin A \cos A} \\
&= \frac{\cos A}{\sin A \cos A} - \frac{\sin A}{\sin A \cos A} \\
&= \frac{1}{\sin A} - \frac{1}{\cos A} \\
&= \operatorname{cosec} A - \sec A \\
&= R.H.S
\end{aligned}$$

Hence proved.

$$64. \frac{\sin A}{\sec A + \tan A - 1} + \frac{\cot A}{\operatorname{cosec} A + \cot A - 1} = 1$$

Sol:

$$\begin{aligned}
LHS &= \frac{\sin A}{\frac{1}{\cos A} + \frac{\sin A}{\cos A} - 1} + \frac{\cos A}{\frac{1}{\sin A} + \frac{\cos A}{\sin A} - 1} \\
&= \frac{\sin A}{\frac{1 + \sin A - \cos A}{\cos A}} + \frac{\cos A}{\frac{1 + \cos A - \sin A}{\sin A}} \\
&= \frac{\sin A \cos A}{1 + \sin A - \cos A} + \frac{\sin A \cos A}{1 + \cos A - \sin A} \\
&= \sin A \cos A \left[\frac{1}{1 + \sin A - \cos A} + \frac{1}{1 + \cos A - \sin A} - \sin A \right]
\end{aligned}$$

$$\begin{aligned}
&= \sin A \cos A \left[\frac{1 + \cos A - \sin A + \cot A \sin A - \cos A}{(1 + \sin \theta - \cos \theta)(1 + \cos A - \sin A)} \right] \\
&= \sin A \cos A \left[\frac{2}{\cos A - \sin A + \sin A + \sin A \cos A - \sin^2 A - \cos A - \cos^2 A + \cos A \sin A} \right] \\
&= \sin A \cos A \left[\frac{2}{1 - \sin^2 A - \cos^2 A + 2 \sin A \cos A} \right] \\
&= \sin A \cos A \left[\frac{2}{1 - (\sin^2 A + \cos^2 A) + 2 \sin A \cos A} \right] \\
&= \sin A \cos A \left[\frac{2}{1 - 1 + 2 \sin A \cos A} \right] (\because \sin^2 A + \cos^2 A = 1) \\
&= \sin A \times \cos A \times \frac{2}{2 \sin A \cos A} \\
&= 1 \\
&L.H.S = R.H.S
\end{aligned}$$

65. $\frac{\tan A}{(1 + \tan^2 A)} + \frac{\cos A}{(1 + \cot^2 A)^2} = \sin A \cos A$

Sol:

$$\begin{aligned}
&= \frac{\tan A}{(\sec^2 A)^2} + \frac{\cos A}{(\operatorname{cosec}^2 A)^2} \quad \left[\begin{array}{l} \because 1 + \tan^2 A = \sec^2 A \\ 1 + \cot^2 A = \operatorname{cosec}^2 A \end{array} \right] \\
&= \frac{\sin A}{\sec^4 A} + \frac{\cot A}{\operatorname{cosec}^4 A} \\
&= \frac{\sin A}{\frac{1}{\cos^4 A}} + \frac{\cos A}{\frac{1}{\sin^4 A}} \\
&= \frac{\sin A}{\cos A} \times \frac{\cos^4 A}{1} + \frac{\cos A}{\sin A} \times \frac{\sin^4 A}{1} \\
&= \sin A \times \cos^3 A + \cos A - \sin^3 \\
&= \sin A \cos A (\cos^2 A + \sin^2 A) \\
&= \sin A \cos A \\
&L.H.S = R.H.S \\
&\text{Hence proved.}
\end{aligned}$$

66. $\sec^4 A(1 - \sin^4 A) - 2 \tan^4 A = 1$

Sol:

$$\begin{aligned}
 LHS &= \sec^4 A(1 - \sin^4 A) - 2 \tan^4 A \\
 &= \sec^4 A - \sec^4 A \times \sin^4 A - 2 \tan^4 A \\
 &= \sec^4 A - \frac{1}{\cos^4 A} \times \sin^4 A - 2 \tan^4 A \\
 &= \sec^4 A - \tan^4 A - 2 \tan^4 A \\
 &= (\sec^2 A)^2 - \tan^4 A - 2 \tan^4 A \\
 &= (1 + \tan^2 A)^2 - \tan^4 A - 2 \tan^4 A \quad \left[\because \sec^2 A - \tan^2 A = 1 \right] \\
 &= 1 + \tan^4 A + 2 \tan^2 A - \tan^4 A - 2 \tan^2 A \\
 &= 1 = RHS
 \end{aligned}$$

Hence proved.

67. $\frac{\cot^2 A(\sec A - 1)}{1 + \sin A} = \sec^2 \left[\frac{1 - \sin A}{1 + \sec A} \right]$

Sol:

$$\begin{aligned}
 &= \frac{\frac{\cos^2 A}{\sin^2 A} \left(\frac{1}{\cos A} - 1 \right)}{1 + \sin A} \\
 &= \frac{\frac{\cos^2 A}{\sin^2 A} \left(\frac{1 - \cos A}{\cos A} \right)}{1 + \sin A} \quad \left[\because \sin^2 A + \cos^2 A = 1 \right] \\
 &= \frac{\frac{(\cos A \times \cos A)}{(1 - \cos^2 A)} \left[\frac{1 - \cos A}{\cos A} \right]}{1 + \sin A} \\
 &= \frac{(\cos A)(1 - \cos A)}{(1 + \cos A)(1 - \cos A)} = \frac{1}{1 + \sin A} \\
 &= \frac{\cos A}{(1 + \cos A)(1 + \sin A)}
 \end{aligned}$$

Solving

$$\begin{aligned}
 RHS &= \sec^2 \left[\frac{1 - \sin A}{1 + \sec A} \right] \\
 &= \frac{1}{\cos^2 A} \left[\frac{1 - \sec A}{1 + \sec A} \right] \\
 &= \frac{1}{\cos^2 A} \left[\frac{1 - \sec A}{\cos A + 1} \right] (\cos A) \\
 &= \frac{(1 - \sin A)}{(\cos A)(\cos A + 1)}
 \end{aligned}$$

By multiplying Nr and Dr with $(1 + \sin A)$

$$\begin{aligned}
 &= \frac{(1 - \sin A)}{(\cos A)(1 + \cos A)} \times \frac{1 + \sin A}{1 + \sin A} \\
 &= \frac{(1)^2 - \sin^2 A}{\cos A(1 + \cos A)(1 + \sin A)} \\
 &= \frac{\cos^2 A}{\cos A(1 + \cos A)(1 + \sin A)} \\
 &= \frac{\cos^2 A}{(1 + \cos A)(1 + \sin A)}
 \end{aligned}$$

$L.H.S = R.H.S$ hence proved.

$$68. (1 + \cot A + \tan A)(\sin A - \cos A) = \frac{\sec A}{\cos \sec^2 A} - \frac{\cos \sec A}{\sec^2 A} = \sin A \tan A - \cos A \cot A$$

Sol:

$$\begin{aligned}
 &(1 + \cot A + \tan A)(\sin A - \cos A) \\
 &\sin A - \cos A + \cot A \sin A - \cot A \cos A + \sin A \tan A - \tan A \cos A \\
 &\sin A - \cos A + \frac{\cos A}{\sin A} \times \sin A - \cot A \cos A + \sin A \tan A - \frac{\sin A}{\cos A} \times \cos A \\
 &\sin A - \cos A + \cos A - \cot A \cos A + \sin A \tan A - \sin A \\
 &= \sin A \cos A \cos A \cot A
 \end{aligned}$$

Solving:

$$\frac{\sec A}{\cos \sec^2 A} - \frac{\cos \sec A}{\sec^2 A}$$

$$\begin{aligned}
 & \frac{\frac{1}{\cos A}}{\frac{1}{\sin^2 A}} - \frac{\frac{1}{\sin A}}{\frac{1}{\cos^2 A}} \\
 & \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A} \\
 & \frac{\sin^3 A - \cos^3 A}{\sin A \cos A} \\
 & = \sin A \times \frac{\sin A}{\cos A} - \cos A \times \frac{\cos A}{\sin A} \\
 & = \sin A \tan A - \cos A \cot A \\
 & L.H.S = R.H.S
 \end{aligned}$$

69. $\sin^2 A \cos^2 B - \cos^2 A \sin^2 B = \sin^2 A - \sin^2 B$

Sol:

$$\begin{aligned}
 LHS &= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B \\
 &= \sin^2 A (1 - \sin^2 B) - (1 - \sin^2 A) (\sin^2 A) \quad (\because \cos^2 A = 1 - \sin^2 A) \\
 &= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B \\
 &= \sin^2 A - \sin^2 B \\
 &R.H.S \text{ Hence Proved.}
 \end{aligned}$$

70. $\frac{\cot A + \tan B}{\cot B + \tan A} = \cot A \tan B$

Sol:

$$\begin{aligned}
 LHS &= \frac{\cot A + \tan B}{\cot B + \tan A} \\
 &= \frac{\frac{\cos A}{\sin A} + \frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B} + \frac{\sin A}{\cos A}} \\
 &= \frac{\cos A \cos B + \sin A \sin B}{\cos A \cos B + \sin A \sin B} \\
 &= \frac{\sin A \cos B}{\cos A \sin B} \\
 &= \cot A \tan B
 \end{aligned}$$



$$\begin{aligned}
 &= \frac{\cos A \cos B + \sin A \sin B}{\sin A \cos B} \times \frac{\cos A \sin B}{\cos A \cos B + \sin A \sin B} \\
 &= \frac{\cos A \sin B}{\sin A \cos B} \\
 &= \cot A \tan B \\
 &= RHS
 \end{aligned}$$

Hence proved

71. $\frac{\tan A + \tan B}{\cot A + \cot B} = \tan A \tan B$

Sol:

$$\begin{aligned}
 LHS &= \frac{\tan A + \tan B}{\cot A + \cot B} \\
 &= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}} \\
 &= \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\cos A \sin B + \cos B \sin A}{\sin A \sin B}} \\
 &= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B} \times \frac{\sin A \sin B}{\cos A \sin B + \cos B \sin A} \\
 &= \frac{\sin A \sin B}{\cos A \cos B} \\
 &= \tan A + \tan B = RHS
 \end{aligned}$$

Hence proved

72. $\cot^2 A \operatorname{cosec}^2 B - \cot^2 B \operatorname{cosec}^2 A = \cot^2 A - \cot^2 B$

Sol:

$$\begin{aligned}
 LHS &= \cot^2 A \operatorname{cosec}^2 B - \cot^2 B \operatorname{cosec}^2 A \\
 &= \cot^2 A (1 + \cot^2 B) - \cot^2 B (1 + \cot^2 A) \quad \left[\because \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta \right] \\
 &= \cot^2 A + \cot^2 A \cot^2 B - \cot^2 B - \cot^2 B \cot^2 A \\
 &= \cot^2 A - \cot^2 B.
 \end{aligned}$$

Hence proved

73. $\tan^2 A \sec^2 B - \sec^2 A \tan^2 B = \tan^2 A - \tan^2 B$

Sol:

$$\begin{aligned}
LHS &= \tan^2 A \sec^2 B - \sec^2 A \tan^2 B \\
&= \tan^2 A + (1 + \tan^2 B) - \sec^2 A (\tan^2 A) \\
&= \tan^2 A + \tan^2 A \tan^2 B - \tan^2 B (1 + \tan^2 A) \quad (\because \sec^2 A = 1 + \tan^2 A) \\
&= \tan^2 A + \tan^2 A \tan^2 B - \tan^2 B - \tan^2 B \tan^2 A \\
&= \tan^2 A - \tan^2 B \\
&= RHS
\end{aligned}$$

74. If $x = a \sec \theta + b \tan \theta$ and $y = a \tan \theta + b \sec \theta$, prove that $x^2 - y^2 = a^2 - b^2$

Sol:

$$\begin{aligned}
L.H.S &= x^2 - y^2 \\
&= (a \sec \theta + b \tan \theta)^2 - (a \tan \theta + b \sec \theta)^2 \\
&= a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta - a^2 \tan^2 \theta - b^2 \sec^2 \theta - 2ab \sec \theta \tan \theta \\
&= a^2 - \sec^2 \theta - b^2 \sec^2 \theta + b^2 \tan^2 \theta - a^2 \tan^2 \theta \\
&= \sec^2 \theta (a^2 - b^2) + \tan^2 \theta (b^2 - a^2) \\
&= \sec^2 \theta (a^2 - b^2) - \tan^2 \theta (a^2 - b^2) \\
&= (a^2 - b^2) (\sec^2 \theta - \tan^2 \theta) \quad [\because \sec^2 \theta - \tan^2 \theta = 1] \\
&= a^2 - b^2
\end{aligned}$$

75. If $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$ and $\frac{x}{a} \sin \theta - \frac{y}{b} \cos \theta = 1$, prove that $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

Sol:

$$\begin{aligned}
\left[\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta \right]^2 + \left[\frac{x}{a} \sin \theta - \frac{y}{b} \cos \theta \right]^2 &= (1)^2 + (1)^2 \\
\frac{x^2}{a^2} \cos^2 \theta + \frac{y^2}{b^2} \sin^2 \theta + \frac{2xy}{ab} \cos \theta \sin \theta + \frac{x^2}{a^2} \sin^2 \theta + \frac{y^2}{b^2} \cos^2 \theta & \\
- \frac{2xy}{ab} \sin \theta \cos \theta &= 1 + 1 \\
\frac{x^2}{a^2} \cos^2 \theta + \frac{y^2}{b^2} \cos^2 \theta + \frac{y^2}{b^2} \sin^2 \theta + \frac{x^2}{a^2} \sin^2 \theta &= 2 \\
\cos^2 \theta \left[\frac{x^2}{a^2} + \frac{y^2}{b^2} \right] + \sin^2 \theta \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} \right) &= 2
\end{aligned}$$

$$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)(\cos^2 \theta + \sin^2 \theta) = 2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \quad (\because \cos^2 \theta + \sin^2 \theta = 1)$$

76. If $\operatorname{cosec} \theta - \sin \theta = a^3$, $\sec \theta - \cos \theta = b^3$, prove that $a^2 b^2 (a^2 + b^2) = 1$

Sol:

$$\operatorname{cosec} \theta - \sin \theta = a^3$$

$$\frac{1}{\sin \theta} - \sin \theta = a^3$$

$$\frac{1 - \sin^2 \theta}{\sin \theta} = a^3$$

$$\frac{\cos^2 \theta}{\sin \theta} = a^3$$

$$a = \frac{\cos^{\frac{1}{3}} \theta}{\sin^{\frac{1}{3}} \theta}$$

$$\Rightarrow a^2 = \frac{\cos^{\frac{4}{3}} \theta}{\sin^{\frac{2}{3}} \theta}$$

$$\sec \theta - \cos \theta = b^3$$

$$\frac{1}{\cos \theta} - \cos \theta = b^3$$

$$\frac{1 - \cos^2 \theta}{\cos \theta} = b^3$$

$$\frac{\sin^2 \theta}{\cos \theta} = b^3$$

$$b = \frac{\sin^{\frac{2}{3}} \theta}{\cos^{\frac{1}{3}} \theta}$$

$$\text{Now, } a^2 b^2 (a^2 + b^2)$$

$$= \frac{\cos^{\frac{4}{3}} \theta}{\sin^{\frac{2}{3}} \theta} \times \frac{\sin^{\frac{4}{3}} \theta}{\cos^{\frac{2}{3}} \theta} \left(\frac{\cos^{\frac{4}{3}} \theta}{\sin^{\frac{2}{3}} \theta} + \frac{\sin^{\frac{4}{3}} \theta}{\cos^{\frac{2}{3}} \theta} \right)$$



$$\begin{aligned}
&= \cos^{\frac{4}{3}-\frac{2}{3}} \theta \times \sin^{\frac{4-2}{3}} \left(\frac{\cos^{\frac{4}{3}} \theta}{\sin^{\frac{2}{3}} \theta} + \frac{\sin^{\frac{4}{3}} \theta}{\cos^{\frac{2}{3}} \theta} \right) \\
&= \cos^{\frac{2}{3}} \theta \sin^{\frac{2}{3}} \left(\frac{1}{\sin^{\frac{2}{3}} \theta \cos^{\frac{2}{3}} \theta} \right) (\because \cos^2 \theta + \sin^2 \theta = 1) \\
&= 1 \\
&L.H.S = R.H.S
\end{aligned}$$

77. If $a \cos^3 \theta + 3a \cos \theta \sin^2 \theta = m$, $a \sin^3 \theta + 3a \cos^2 \theta \sin \theta = n$, prove that

$$(m+n)^{\frac{2}{3}} + (m-n)^{\frac{2}{3}}$$

Sol:

$$\begin{aligned}
&= (a \cos^3 \theta + 3a \cos \theta \sin^2 \theta + a \sin^3 \theta + 3a \cos^2 \theta \sin \theta)^{\frac{2}{3}} \\
&+ (a \cos^3 \theta + 3a \cos \theta \sin^2 \theta - a \sin^3 \theta - 3a \cos^2 \theta \sin \theta)^{\frac{2}{3}} \\
&= a^{\frac{1}{3}} (\cos^3 \theta + 3 \cos \theta \sin^2 \theta + \sin^3 \theta + 3 \cos^2 \theta \sin \theta)^{\frac{2}{3}} \\
&+ a^{\frac{2}{3}} (\cos^3 \theta + 3 \cos \theta \sin^2 \theta + \sin^3 \theta - 3 \cos^2 \theta \sin \theta)^{\frac{2}{3}} \\
&= a^{\frac{1}{3}} \left[(\cos \theta + \sin \theta)^3 \right]^{\frac{2}{3}} + a^{\frac{2}{3}} (\cos \theta - \sin \theta)^3 \left]^{\frac{2}{3}} \\
&= a^{\frac{2}{3}} \left[(\cos \theta + \sin \theta)^2 \right] + a^{\frac{2}{3}} (\cos \theta - \sin \theta)^2 \\
&= a^{\frac{2}{3}} [\cos^2 \theta + \sin^2 \theta - 2 \sin \theta \cos \theta] \\
&= a^{\frac{2}{3}} [\cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta] + a^{\frac{2}{3}} [\cos^2 \theta + \sin^2 \theta - 2 \sin \theta \cos \theta] \\
&= a^{\frac{2}{3}} [1 + 2 \sin \theta \cos \theta] + a^{\frac{2}{3}} [1 - 2 \sin \theta \cos \theta] \\
&= a^{\frac{2}{3}} [1 + 2 \sin \theta \cos \theta + 1 - 2 \sin \theta \cos \theta] \\
&= a^{\frac{1}{3}} (1+1) = 2a^{\frac{2}{3}} \\
&= R.H.S
\end{aligned}$$

Hence proved.

78. If $x = a \cos^3 \theta$, $y = b \sin^3 \theta$, prove that $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$

Sol:

$$x = a \cos^3 \theta : y = b \sin^3 \theta$$

$$\frac{x}{a} = \cos^3 \theta : \frac{y}{b} = \sin^3 \theta$$

$$L.H.S = \left[\frac{x}{a}\right]^{\frac{2}{3}} + \left[\frac{y}{b}\right]^{\frac{2}{3}}$$

$$= (\cos^3 \theta)^{\frac{2}{3}} + (\sin^3 \theta)^{\frac{2}{3}}$$

$$= \cos^2 \theta + \sin^2 \theta \quad (\because \cos^2 \theta + \sin^2 \theta = 1)$$

$$= 1$$

Hence proved

79. If $3 \sin \theta + 5 \cos \theta = 5$, prove that $5 \sin \theta - 3 \cos \theta = \pm 3$.

Sol:

$$\text{Given } 3 \sin \theta + 5 \cos \theta = 5$$

$$3 \sin \theta = 5 - 5 \cos \theta$$

$$3 \sin \theta = 5(1 - \cos \theta)$$

$$3 \sin \theta = \frac{5(1 - \cos \theta)(1 - \cos \theta)}{1 + \cos \theta}$$

$$3 \sin \theta = \frac{5(1 - \cos^2 \theta)}{(1 + \cos \theta)}$$

$$3 \sin \theta = \frac{5 \sin^2 \theta}{1 + \cos \theta}$$

$$3 + 3 \cos \theta = 5 \sin \theta$$

$$3 = 5 \sin \theta - 3 \cos \theta$$

$$= RHS$$

Hence proved.

80. If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$, prove that $a^2 + b^2 = m^2 + n^2$

Sol:

$$R.H.S = m^2 \sin^2$$

$$= (a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2$$

$$= a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \sin \theta \cos \theta$$

$$+ a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta$$

$$= a^2 \cos^2 \theta + b^2 \cos^2 \theta + b^2 \sin^2 \theta + a^2 \sin^2 \theta$$

$$= a^2 (\sin^2 \theta + \cos^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta)$$

$$= a^2 + b^2 \quad (\because \sin^2 \theta + \cos^2 \theta = 1)$$

81. If $\cos \theta + \cot \theta = m$ and $\operatorname{cosec} \theta - \cot \theta = n$, prove that $m n = 1$

Sol:

$$LHS = mn$$

$$= (\operatorname{cosec} \theta + \cot \theta)(\operatorname{cosec} \theta - \cot \theta)$$

$$= \operatorname{cosec}^2 \theta - \cot^2 \theta$$

$$= 1 \quad \left[\because (a+b)(a-b) = a^2 - b^2 \operatorname{cosec}^2 \theta - \cot^2 \theta = 1 \right]$$

$$= R.H.S$$

82. If $\cos A + \cos^2 A = 1$, prove that $\sin^2 A + \sin^4 A = 1$

Sol:

$$\cos A + \cos^2 A = 1$$

$$\cos A = 1 - \cos^2 A$$

$$\cos A = \sin^2 A$$

$$LHS = \sin^2 A + \sin^4 A$$

$$= \sin^2 A + (\sin^2 A)$$

$$= \sin^2 A + (\cos A)^2$$

$$= \sin^2 A + \cos A$$

$$= 1$$

83. Prove that:

$$(i) \sqrt{\frac{\sec \theta - 1}{\sec \theta + 1}} + \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}} = 2 \operatorname{cosec} \theta$$

$$(ii) \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} + \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} = 2 \sec \theta$$

$$(iii) \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} + \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = 2 \operatorname{cosec} \theta$$

$$(iv) \frac{\sec \theta - 1}{\sec \theta + 1} = \left(\frac{\sin \theta}{1 + \cos \theta} \right)^2$$

Sol:

$$LHS = \sqrt{\frac{\frac{1}{\cos \theta} - 1}{\frac{1}{\cos \theta} + 1}} + \sqrt{\frac{\frac{1}{\cos \theta} + 1}{\frac{1}{\cos \theta} - 1}}$$

$$\begin{aligned}
&= \sqrt{\frac{1-\cos\theta}{\cos\theta}} + \sqrt{\frac{1+\cos\theta}{\cos\theta}} \\
&= \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} + \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} \\
&= \sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)} \times \frac{(1-\cos\theta)}{1-\cos\theta}} + \sqrt{\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}} \\
&= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}} + \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} \\
&= \frac{1-\cos\theta}{\sin\theta} + \frac{1+\cos\theta}{\sin\theta} \\
&= \frac{1-\cos\theta+1+\cos\theta}{\sin\theta} \\
&= \frac{2}{\sin\theta} \\
&= 2\operatorname{cosec}\theta
\end{aligned}$$

$$\begin{aligned}
(2) \quad &\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} \\
&\sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{(1+\sin\theta)}{1+\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}} \\
&\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} \\
&= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} + \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}} \\
&= \frac{1+\cos\theta}{\sin\theta} + \frac{1-\cos\theta}{\sin\theta} \\
&= \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta
\end{aligned}$$

(3) Not given

$$\begin{aligned}
 (4) & \frac{\sec \theta - 1}{\sec \theta + 1} \\
 &= \frac{\frac{1}{\cos \theta} - 1}{\frac{1}{\cos \theta} + 1} \\
 &= \frac{1 - \cos \theta}{1 + \cos \theta} \\
 &= \frac{1 - \cos \theta}{1 + \cos \theta} \times \frac{1 + \cos \theta}{1 + \cos \theta} \\
 &= \frac{1 - \cos^2 \theta}{(1 + \cos \theta)^2} \\
 &= \frac{\sin^2 \theta}{(1 + \cos \theta)^2} \\
 &= \left[\frac{\sin \theta}{1 + \cos \theta} \right]^2 \\
 &= RHS
 \end{aligned}$$

Hence proved.

84. If $\cos \theta + \cos^2 \theta = 1$, prove that

$$\sin^{12} \theta + 3\sin^{10} \theta + 3\sin^8 \theta + \sin^6 \theta + 2\sin^4 \theta + 2\sin^2 \theta - 2 = 1$$

Sol:

$$\cos \theta + \cos^2 \theta = 1$$

$$\cos \theta = 1 - \cos^2 \theta$$

$$\cos \theta = \sin^2 \theta \quad \dots (1)$$

$$\text{Now, } \sin^{12} \theta + 3\sin^{10} \theta + 3\sin^8 \theta + \sin^6 \theta + 2\sin^4 \theta + 2\sin^2 \theta - 2$$

$$= (\sin^4 \theta)^3 + 3\sin^4 \theta \cdot \sin^2 \theta (\sin^4 \theta + \sin^2 \theta)$$

$$+ (\sin^2 \theta)^3 + 2(\sin^2 \theta)^2 + 2\sin^2 \theta$$

Using $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$ and also from

$$(1) \sin^2 \theta \cos \theta$$

$$(\sin^4 \theta + \sin^2 \theta)^3 + 2\cos^2 \theta + 2\cos \theta - 2.$$

$$\left((\sin^2 \theta)^2 + \sin^2 \theta \right) + 2\cos^2 \theta + 2\cos \theta - 2$$

$$(\cos^2 + \sin^2 \theta)^3 + 2\cos^2 \theta + 2\cos \theta - 2$$

$$(\cos^2 + \sin^2)^3 + 2\cos^2 \theta + 2\sin^2 \theta - 2 \quad \left[\because \sin^2 \theta + \cos^2 \theta = 1 \right]$$

$$1 + 2(\sin^2 \theta + \cos^2 \theta) - 2$$

$$1 + 2(1) - 2$$

$$= 1$$

$$L.H.S = R.H.S$$

Hence proved.

85. Given that $(1 + \cos \alpha)(1 + \cos \beta)(1 + \cos \gamma) = (1 - \cos \alpha)(1 - \cos \beta)(1 - \cos \gamma)$

Show that one of the values of each member of this equality is $\sin \alpha \sin \beta \sin \gamma$

Sol:

L.H.S

$$\text{We know that } 1 + \cos \theta = 1 + \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} = 2\cos^2 \frac{\theta}{2}$$

$$\therefore \Rightarrow 2\cos^2 \frac{\alpha}{2} \cdot 2\cos^2 \frac{\beta}{2} \cdot 2\cos^2 \frac{\gamma}{2} \quad \dots(1)$$

Multiply (1) with $\sin \alpha \sin \beta \sin \gamma$ and divide it with same we get

$$\frac{8\cos^2 \frac{\alpha}{2} \cos^2 \frac{\beta}{2} \cos^2 \frac{\gamma}{2}}{\sin \alpha \sin \beta \sin \gamma} \times \sin \alpha \sin \beta \sin \gamma$$

$$\Rightarrow \frac{2\cos^2 \frac{\alpha}{2} \cos^2 \frac{\beta}{2} \cos^2 \frac{\gamma}{2} \times \sin \alpha \sin \beta \sin \gamma}{\sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2}}$$

$$\Rightarrow \sin \alpha \sin \beta \sin \gamma \times \cot \frac{\alpha}{2} \cot \frac{\beta}{2} \cot \frac{\gamma}{2}$$

$$RHS (1 - \cos \alpha)(1 - \cos \beta)(1 - \cos \gamma)$$

$$\text{We know that } 1 - \cos \theta = 1 - \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} = 2\sin^2 \frac{\theta}{2}$$

$$\Rightarrow 2\sin^2 \frac{\alpha}{2} \cdot 2\sin^2 \frac{\beta}{2} \cdot 2\sin^2 \frac{\gamma}{2}$$

Multiply and divide by $\sin \alpha \sin \beta \sin \gamma$ we get

$$\frac{2\sin^2 \frac{\alpha}{2} 2\sin^2 \frac{\beta}{2} 2\sin^2 \frac{\gamma}{2} \cdot \sin \alpha \sin \beta \sin \gamma}{\sin \alpha \sin \beta \sin \gamma}$$

$$\Rightarrow \frac{2 \sin^2 \frac{\alpha}{2} \cdot 2 \sin^2 \frac{\beta}{2} \cdot 2 \sin^2 \frac{\gamma}{2} \cdot \sin \alpha \sin \beta \sin \gamma}{2 \sin \frac{\alpha}{2} \cos \frac{\beta}{2} 2 \sin \frac{\beta}{2} \cos \frac{\beta}{2} 2 \sin \frac{\gamma}{2} \cos \frac{\gamma}{2}}$$

$$\Rightarrow \tan \frac{\alpha}{2} \tan \frac{\beta}{2} \tan \frac{\gamma}{2} \sin \alpha \sin \beta \sin \gamma$$

Hence $\sin \alpha \sin \beta \sin \gamma$ is the member of equality.

86. if $\sin \theta + \cos \theta = x$ P.T $\sin^6 \theta + \cos^6 \theta = \frac{4 - 3(x^2 - 1)^2}{4}$

Sol:

$$\sin \theta + \cos \theta = x$$

Squaring on both sides

$$(\sin \theta + \cos \theta)^2 = x^2$$

$$\Rightarrow \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = x^2$$

$$\therefore \sin \theta \cos \theta = \frac{x^2 - 1}{2} \quad \dots\dots(1)$$

$$\text{We know } \sin^2 \theta + \cos^2 \theta = 1$$

Cubing on both sides

$$(\sin^2 \theta + \cos^2 \theta)^3 = (1)^3$$

$$\sin^6 \theta + \cos^6 \theta + 3 \sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta) = 1$$

$$\Rightarrow \sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cos^2 \theta$$

$$= 1 - 3 \frac{(x^2 - 1)^2}{4} \text{ from (1)}$$

$$\therefore \sin^6 \theta + \cos^6 \theta = \frac{4 - 3(x^2 - 1)^2}{4}$$

Hence proved

87. if $x = a \sec \theta \cos \phi$, $y = b \sec \theta \sin \phi$ and $z = c \tan \theta$, S.T $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

Sol:

$$x^2 = a^2 \sec^2 \theta \cos^2 \theta \quad \dots\dots(i)$$

$$y^2 = b^2 \sec^2 \theta \sin^2 \theta \quad \dots\dots(ii)$$

$$z^2 = c^2 \tan^2 \theta \quad \dots\dots(iii)$$

Exercise 6.2

1. If $\cos \theta = \frac{4}{5}$, find all other trigonometric ratios of angle θ

Sol:

$$\text{We have } \sin \theta = \sqrt{1 - \cos^2 \theta} = \sqrt{1 - \left(\frac{4}{5}\right)^2}$$

$$= \sqrt{1 - \frac{16}{25}}$$

$$= \sqrt{25 - \frac{16}{25}}$$

$$= \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\therefore \sin \theta = \frac{3}{5}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{3/5}{4/5} = \frac{3}{4} \quad \sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3} \quad \cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}.$$

2. If $\sin \theta = \frac{1}{\sqrt{2}}$, find all other trigonometric ratios of angle θ

Sol:

$$\text{We have } \cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \sqrt{\frac{1-1}{2}} = \sqrt{\frac{1}{2}}$$

$$\therefore \cos \theta = \frac{1}{\sqrt{2}}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{1} = 1$$

3. If $\tan \theta = \frac{1}{\sqrt{2}}$, Find the value of $\frac{\cos \sec^2 \theta - \sec^2 \theta}{\cos \sec^2 \theta + \cot^2 \theta}$

Sol:

We know that $\sec \theta = \sqrt{1 + \tan^2 \theta}$

$$= \sqrt{1 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \sqrt{1 + \frac{1}{2}} = \sqrt{\frac{3}{2}}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\cos \sec \theta = \sqrt{1 + \cot^2 \theta} \Rightarrow \sqrt{1 + 2} = \sqrt{3}$$

Substituting it in (1) we get

$$\begin{aligned} \Rightarrow \frac{(\sqrt{3})^2 - (\sqrt{3})^2}{(\sqrt{3})^2 + (\sqrt{3})^2} &= \frac{3 - \frac{3}{2}}{3 + 2} = \frac{\frac{3}{2}}{5} \\ &= \frac{3}{10} \end{aligned}$$

4. If $\tan \theta = \frac{3}{4}$, find the value of $\frac{1 - \cos \theta}{1 + \cos \theta}$

Sol:

$$\sec \theta = \sqrt{1 + \tan^2 \theta} = \sqrt{1 + \left(\frac{3}{4}\right)^2} = \sqrt{1 + \frac{9}{16}}$$

$$\Rightarrow \sqrt{\frac{16 + 9}{16}} = \frac{5}{4}$$

$$\therefore \sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4} = \cos \theta$$



$$\therefore \text{We get } \frac{1 - \frac{4}{5}}{1 + \frac{4}{5}} = \frac{\frac{1}{5}}{\frac{9}{5}} = \frac{1}{9}.$$

5. If $\tan \theta = \frac{12}{5}$, find the value of $\frac{1 + \sin \theta}{1 - \sin \theta}$

Sol:

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$$

$$\operatorname{cosec} \theta = \sqrt{1 + \cot^2 \theta} = \sqrt{1 + \left[\frac{5}{12}\right]^2} = \sqrt{\frac{144 + 25}{(12)^2}} = \sqrt{\frac{169}{144}} = \frac{13}{12}.$$

$$\sin \theta = \frac{1}{\operatorname{cosec} \theta} = \frac{1}{\frac{13}{12}} = \frac{12}{13}.$$

$$\text{We get } \frac{1 + \frac{12}{13}}{1 - \frac{12}{13}} = \frac{\frac{13 + 12}{13}}{\frac{13 - 12}{13}} = \frac{25}{1} = 25$$

6. If $\cot \theta = \frac{1}{\sqrt{3}}$, find the value of $\frac{1 - \cos^2 \theta}{2 - \sin^2 \theta}$

Sol:

$$\operatorname{cosec} \theta = \sqrt{1 + \cot^2 \theta} = \sqrt{1 + \frac{1}{3}} = \sqrt{\frac{4}{3}}$$

$$\therefore \operatorname{cosec} \theta = \frac{2}{\sqrt{3}}$$

$$\sin \theta = \frac{1}{\operatorname{cosec} \theta} = \frac{1}{\frac{2}{\sqrt{3}}} = \frac{\sqrt{3}}{2}$$

$$\text{and } \frac{1}{\cot \theta} = \frac{\sin \theta}{\cos \theta} = \cos \theta = \frac{\sin \theta}{\frac{1}{\cos \theta}} \Rightarrow \frac{\frac{\sqrt{3}}{2}}{\frac{2}{\sqrt{3}}} = \frac{1}{2}.$$

$\therefore$ on substituting we get

$$\frac{1 - \frac{1}{4}}{2 - \frac{3}{4}} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5}.$$

7. If $\operatorname{cosec} A = \sqrt{2}$, find the value of $\frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)}$

Sol:

We know that $\cot A = \sqrt{\operatorname{cosec}^2 A - 1}$

$$= \sqrt{(2)^2 - 1} = \sqrt{2 - 1}$$

-1 .

$$\tan A = \frac{1}{\cot A} = \frac{1}{1} = 1$$

$$\sin A = \frac{1}{\operatorname{cosec} A} = \frac{1}{\sqrt{2}} \therefore \sin A = \frac{1}{\sqrt{2}}$$

$$\cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}.$$

On substituting we get

$$\frac{2\left[\frac{1}{\sqrt{2}}\right]^3 + 3[1]^2}{4\left[1 - \left(\frac{1}{\sqrt{2}}\right)^2\right]} = \frac{2 = \frac{1}{2} + 3}{4\left[1 - \frac{1}{2}\right]}$$

$$\Rightarrow \frac{1 + 3}{4 \cdot \frac{1}{2}} = \frac{4}{2} = 2.$$

8. If $\cot \theta = \sqrt{3}$, find the value of $\frac{\operatorname{cosec}^2 \theta + \cot^2 \theta}{\operatorname{cosec}^2 \theta - \cot^2 \theta}$

Sol:

$$\operatorname{cosec} \theta = \sqrt{1 + \cot^2 \theta} = \sqrt{1 + (\sqrt{3})^2} = \sqrt{1 + 3} = 2$$

$$\sin \theta = \frac{1}{\operatorname{cosec} \theta} = \frac{1}{2} \cot \theta = \frac{\cos \theta}{\sin \theta} \therefore \cos \theta = \cot \theta \cdot \sin \theta$$

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{2}{\sqrt{3}}$$

On substituting we get

$$\frac{(2)^2 + (\sqrt{3})^2}{(2)^2 - \left(\frac{2}{\sqrt{3}}\right)^2} = \frac{4+3}{\frac{12-4}{3}} = \frac{7}{\frac{8}{3}} = \frac{21}{8}.$$

9. If $3\cos \theta = 1$, find the value of $\frac{6\sin^2 \theta + \tan^2 \theta}{4\cos \theta}$

Sol:

$$\cos \theta = \frac{1}{3} \quad \sin = \sqrt{1 + \cos^2 \theta}$$

$$= \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{2\sqrt{2}}{3 \cdot \frac{1}{3}} = 2\sqrt{2}$$

On substituting in (1) we get

$$\frac{6 \left[\frac{2\sqrt{2}}{3} \right]^2 + (2\sqrt{2})^2}{4 \cdot \frac{1}{3}} = \frac{6 \cdot \frac{3}{5}}{\frac{4}{5}} = \frac{16+24}{\frac{4}{3}} = \frac{40}{4} = 10$$

10. If $\sqrt{3} \tan \theta = \sin \theta$, find the value of $\sin^2 \theta - \cos^2 \theta$

Sol:

$$\sqrt{3} \cdot \frac{\sin \theta}{\cos \theta} = \sin \theta$$

$$\cos \theta = \frac{\sqrt{3}}{3} \Rightarrow \frac{1}{\sqrt{3}}$$

$$\begin{aligned}\sin \theta &= \sqrt{1 - \cos^2 \theta} = \sqrt{1 - \left(\frac{1}{\sqrt{3}}\right)^2} \\ \therefore \sin^2 \theta - \cos^2 \theta &= \left(\sqrt{\frac{2}{3}}\right)^2 - \left[\frac{1}{\sqrt{3}}\right]^2 \\ &= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}\end{aligned}$$

11. If $\operatorname{cosec} \theta = \frac{13}{12}$, find the value of $\frac{2\sin \theta - 3\cos \theta}{4\sin \theta - 9\cos \theta}$

Sol:

$$\begin{aligned}\sin \theta &= \frac{1}{\operatorname{cosec} \theta} = \frac{1}{\frac{13}{12}} = \frac{12}{13} \\ \cos \theta &= \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \left[\frac{12}{13}\right]^2} = \sqrt{1 - \frac{144}{169}} \\ &= \sqrt{\frac{25}{169}} = \frac{5}{13} \\ \Rightarrow \frac{2 \cdot \frac{12}{13} - 3 \cdot \frac{5}{13}}{4 \cdot \frac{12}{13} - 9 \cdot \frac{5}{13}} &= \frac{24 - 15}{48 - 15} = \frac{9}{3} = 3\end{aligned}$$

12. If $\sin \theta + \cos \theta = \sqrt{2} \cos(90^\circ - \theta)$, find $\cot \theta$ find $\cot \theta$

Sol:

$$L.H.S \Rightarrow \sin \theta + \cos \theta = \sqrt{2} \sin \theta \quad \left[\because \cos(90 - \theta) = \sin \theta \right]$$

$$\Rightarrow \cos \theta - \sin \theta (\sqrt{2}) = \sin \theta$$

$$\cos \theta - \sin \theta (\sqrt{2} - 1)$$

Divide both sides with $\sin \theta$ we get

$$\frac{\cos \theta}{\sin \theta} = \frac{\sin \theta}{\sin \theta} (\sqrt{2} - 1)$$

$$= \cot \theta = \sqrt{2} - 1$$

