

Exercise – 9.1

1. Write the first terms of each of the following sequences whose n^{th} term are

(i) $a_n = 3n + 2$

(ii) $a_n = \frac{n-2}{3}$

(iii) $a_n = 3^n$

(iv) $a_n = \frac{3n-2}{5}$

(v) $a_n = (-1)^n 2^n$

(vi) $a_n = \frac{n(n-2)}{2}$

(vii) $a_n = n^2 - n + 1$

(viii) $a_n = n^2 - n + 1$

(ix) $a_n = \frac{2n-3}{6}$

Sol:

We have to write first five terms of given sequences

(i) $a_n = 3n + 2$

Given sequence $a_n = 3n + 2$

To write first five terms of given sequence put $n = 1, 2, 3, 4, 5$, we get

$$a_1 = (3 \times 1) + 2 = 3 + 2 = 5$$

$$a_2 = (3 \times 2) + 2 = 6 + 2 = 8$$

$$a_3 = (3 \times 3) + 2 = 9 + 2 = 11$$

$$a_4 = (3 \times 4) + 2 = 12 + 2 = 14$$

$$a_5 = (3 \times 5) + 2 = 15 + 2 = 17$$

$\therefore$ The required first five terms of given sequence $a_n = 3n + 2$ are 5, 8, 11, 14, 17.

(ii) $a_n = \frac{n-2}{3}$

Given sequence $a_n = \frac{n-2}{3}$

To write first five terms of given sequence $a_n = \frac{n-2}{3}$

put $n = 1, 2, 3, 4, 5$ then we get

$$a_1 = \frac{1-2}{3} = \frac{-1}{3}; a_2 = \frac{2-2}{3} = 0$$

$$a_3 = \frac{3-2}{3} = \frac{1}{3}; a_4 = \frac{4-2}{3} = \frac{2}{3}$$

$$a_5 = \frac{5-2}{3} = 1$$

$\therefore$ The required first five terms of given sequence $a_n = \frac{n-2}{3}$ are $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, 1$.

(iii) $a_n = 3^n$

Given sequence $a_n = 3^n$

To write first five terms of given sequence, put $n = 1, 2, 3, 4, 5$ in given sequence.

Then,

$$a_1 = 3^1 = 3; a_2 = 3^2 = 9; a_3 = 3^3 = 27; a_4 = 3^4 = 81; a_5 = 3^5 = 243.$$

(iv) $a_n = \frac{3n-2}{5}$

Given sequence, $a_n = \frac{3n-2}{5}$

To write first five terms, put $n = 1, 2, 3, 4, 5$ in given sequence $a_n = \frac{3n-2}{5}$

Then, we get

$$a_1 = \frac{3 \times 1 - 2}{5} = \frac{3-2}{5} = \frac{1}{5}$$

$$a_2 = \frac{3 \times 2 - 2}{5} = \frac{6-2}{5} = \frac{4}{5}$$

$$a_3 = \frac{3 \times 3 - 2}{5} = \frac{9-2}{5} = \frac{7}{5}$$

$$a_4 = \frac{3 \times 4 - 2}{5} = \frac{12-2}{5} = \frac{10}{5}$$

$$a_5 = \frac{3 \times 5 - 2}{5} = \frac{15-2}{5} = \frac{13}{5}$$

$\therefore$ The required first five terms are $\frac{1}{5}, \frac{4}{5}, \frac{7}{5}, \frac{10}{5}, \frac{13}{5}$

(v) $a_n = (-1)^n 2^n$

Given sequence is $a_n = (-1)^n 2^n$

To get first five terms of given sequence a_n , put $n = 1, 2, 3, 4, 5$.

$$a_1 = (-1)^1 \cdot 2^1 = (-1) \cdot 2 = -2$$

$$a_2 = (-1)^2 \cdot 2^2 = (-1) \cdot 4 = 4$$

$$a_3 = (-1)^3 \cdot 2^3 = (-1) \cdot 8 = -8$$

$$a_4 = (-1)^4 \cdot 2^4 = (-1) \cdot 16 = 16$$

$$a_5 = (-1)^5 \cdot 2^5 = (-1) \cdot 32 = -32$$

$\therefore$ The first five terms are -2, 4, -8, 16, -32.

(vi) $a_n = \frac{n(n-2)}{2}$

The given sequence is, $a_n = \frac{n(n-2)}{2}$

To write first five terms of given sequence $a_n = \frac{n(n-2)}{2}$

Put $n = 1, 2, 3, 4, 5$. Then, we get

$$a_1 = \frac{1(1-2)}{2} = \frac{1-1}{2} = \frac{-1}{2}$$

$$a_2 = \frac{2(2-2)}{2} = \frac{2 \cdot 0}{2} = 0$$

$$a_3 = \frac{3(3-2)}{2} = \frac{3 \cdot 1}{2} = \frac{3}{2}$$

$$a_4 = \frac{4(4-2)}{2} = \frac{4 \cdot 2}{2} = 4$$

$$a_5 = \frac{5(5-2)}{2} = \frac{5 \cdot 3}{2} = \frac{15}{2}$$

$\therefore$ The required first five terms are $\frac{-1}{2}, 0, \frac{3}{2}, 4, \frac{15}{2}$.

(vii) $a_n = n^2 - n + 1$

The given sequence is, $a_n = n^2 - n + 1$

To write first five terms of given sequence a_n we get put $n = 1, 2, 3, 4, 5$. Then we get $a_1 = 1^2 - 1 + 1 = 1$

$$a_2 = 2^2 - 2 + 1 = 3$$

$$a_3 = 3^2 - 3 + 1 = 7$$

$$a_4 = 4^2 - 4 + 1 = 13$$

$$a_5 = 5^2 - 5 + 1 = 21$$

$\therefore$ The required first five terms of given sequence $a_n = n^2 - n + 1$ are 1, 3, 7, 13, 21

(viii) $a_n = 2n^2 - 3n + 1$

The given sequence is $a_n = 2n^2 - 3n + 1$

To write first five terms of given sequence a_n , we put $n = 1, 2, 3, 4, 5$. Then we get

$$a_1 = 2 \cdot 1^2 - 3 \cdot 1 + 1 = 2 - 3 + 1 = 0$$

$$a_2 = 2 \cdot 2^2 - 3 \cdot 2 + 1 = 8 - 6 + 1 = 3$$

$$a_3 = 2 \cdot 3^2 - 3 \cdot 3 + 1 = 18 - 9 + 1 = 10$$

$$a_4 = 2 \cdot 4^2 - 3 \cdot 4 + 1 = 32 - 12 + 1 = 21$$

$$a_5 = 2 \cdot 5^2 - 3 \cdot 5 + 1 = 50 - 15 + 1 = 36$$

$\therefore$ The required first five terms of given sequence $a_n = 2n^2 - 3n + 1$ are 0, 3, 10, 21, 36

(ix) $a_n = \frac{2n-3}{6}$

Given sequence is, $a_n = \frac{2n-3}{6}$

To write first five terms of given sequence we put $n = 1, 2, 3, 4, 5$. Then, we get,

$$a_1 = \frac{2 \cdot 1 - 3}{6} = \frac{2 - 3}{6} = \frac{-1}{6}$$

$$a_2 = \frac{2 \cdot 2 - 3}{6} = \frac{4 - 3}{6} = \frac{1}{6}$$

$$a_3 = \frac{2 \cdot 3 - 3}{6} = \frac{6 - 3}{6} = \frac{3}{6} = \frac{1}{2}$$

$$a_4 = \frac{2 \cdot 4 - 3}{6} = \frac{8 - 3}{6} = \frac{5}{6}$$

$$a_5 = \frac{2 \cdot 5 - 3}{6} = \frac{10 - 3}{6} = \frac{7}{6}$$

$\therefore$ The required first five terms of given sequence $a_n = \frac{2n-3}{6}$ are $\frac{-1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{7}{6}$.

2. Find the indicated terms in each of the following sequences whose n th terms are:

- (i) $a_n = 5n - 4$; a_{12} and a_{15}
- (ii) $a_n = \frac{3n-2}{4n+5}$; a_7 and a_8
- (iii) $a_n = n(n-1)(n-2)$; a_5 and a_8
- (iv) $a_n = (n-1)(2-n)(3+n)$; a_{11} a_{21} a_3
- (v) $a_n = (-1)^n n$; a_3, a_5, a_8

Sol:

We have to find the required term of a sequence when n^{th} term of that sequence is given.

- (i) $a_n = 5n - 4$; a_{12} and a_{15}

Given n^{th} term of a sequence $a_n = 5n - 4$

To find 12^{th} term, 15^{th} terms of that sequence, we put $n = 12, 15$ in its n^{th} term.

Then, we get

$$a_{12} = 5 \cdot 12 - 4 = 60 - 4 = 56$$

$$a_{15} = 5 \cdot 15 - 4 = 75 - 4 = 71$$

$\therefore$ The required terms $a_{12} = 56, a_{15} = 71$

- (ii) $a_n = \frac{3n-2}{4n+5}$; a_7 and a_8

Given n^{th} term is $(a_n) = \frac{3n-2}{4n+5}$

To find $7^{\text{th}}, 8^{\text{th}}$ terms of given sequence, we put $n = 7, 8$.

$$a_7 = \frac{(3 \cdot 7) - 2}{(4 \cdot 7) + 5} = \frac{19}{33}$$

$$a_8 = \frac{(3 \cdot 8) - 2}{(4 \cdot 8) + 5} = \frac{22}{37}$$

$\therefore$ The required terms $a_7 = \frac{19}{33}$ and $a_8 = \frac{22}{37}$.

- (iii) $a_n = n(n-1)(n-2)$; a_5 and a_8

Given n^{th} term is $a_n = n(n-1)(n-2)$

To find $5^{\text{th}}, 8^{\text{th}}$ terms of given sequence, put $n = 5, 8$ in an then, we get

$$a_5 = 5(5-1)(5-2) = 5 \cdot 4 \cdot 3 = 60$$

$$a_8 = 8(8-1)(8-2) = 8 \cdot 7 \cdot 6 = 336$$

$\therefore$ The required terms are $a_5 = 60$ and $a_8 = 336$

- (iv) $a_n = (n-1)(2-n)(3+n)$; a_{11} a_{21} a_3

The given n^{th} term is $a_n = (n-1)(2-n)(3+n)$

To find a_1, a_2, a_3 of given sequence put $n = 1, 2, 3$ is an

$$a_1 = (1-1)(2-1)(3+1) = 0 \cdot 1 \cdot 4 = 0$$

$$a_2 = (2-1)(2-2)(3+2) = 1 \cdot 0 \cdot 5 = 0$$

$$a_3 = (3-1)(2-3)(3+3) = 2 \cdot -1 \cdot 6 = -12$$

$\therefore$ The required terms $a_1 = 0, a_2 = 0, a_3 = -12$

- (v) $a_n = (-1)^n n$; a_3, a_5, a_8

The given n^{th} term is, $a_n = (-1)^n \cdot n$

To find a_3, a_5, a_8 of given sequence put $n = 3, 5, 8$, in a_n .

$$a_3 = (-1)^3 \cdot 3 = -1 \cdot 3 = -3$$

$$a_5 = (-1)^5 \cdot 5 = -1 \cdot 5 = -5$$

$$a_8 = (-1)^8 \cdot 8 = 1 \cdot 8 = 8$$

$\therefore$ The required terms $a_3 = -3, a_5 = -5, a_8 = 8$

3. Find the next five terms of each of the following sequences given by:

(i) $a_1 = 1, a_n = a_{n-1} + 2, n \geq 2$

(ii) $a_1 = a_2 = 2, a_n = a_{n-1} - 3, n > 2$

(iii) $a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$

(iv) $a_1 = 4, a_n = 4a_{n-1} + 3, n > 1$

Sol:

We have to find next five terms of following sequences.

(i) $a_1 = 1, a_n = a_{n-1} + 2, n \geq 2$

Given, first term (a_1) = 1,

n^{th} term $a_n = a_{n-1} + 2, n \geq 2$

To find 2nd, 3rd, 4th, 5th, 6th terms, we use given condition $n \geq 2$ for n^{th} term $a_n = a_{n-1} + 2$

$$a_2 = a_{2-1} + 2 = a_1 + 2 = 1 + 2 = 3 (\because a_1 = 1)$$

$$a_3 = a_{3-1} + 2 = a_2 + 2 = 3 + 2 = 5$$

$$a_4 = a_{4-1} + 2 = a_3 + 2 = 5 + 2 = 7$$

$$a_5 = a_{5-1} + 2 = a_4 + 2 = 7 + 2 = 9$$

$$a_6 = a_{6-1} + 2 = a_5 + 2 = 9 + 2 = 11$$

$\therefore$ The next five terms are,

$$a_2 = 3, a_3 = 5, a_4 = 7, a_5 = 9, a_6 = 11$$

(ii) $a_1 = a_2 = 2, a_n = a_{n-1} - 3, n > 2$

Given,

First term (a_1) = 2

Second term (a_2) = 2

n^{th} term (a_n) = $a_{n-1} - 3$

To find next five terms i.e., a_3, a_4, a_5, a_6, a_7 we put $n = 3, 4, 5, 6, 7$ in a_n

$$a_3 = a_{3-1} - 3 = a_2 - 3 = 2 - 3 = -1$$

$$a_4 = a_{4-1} - 3 = a_3 - 3 = -1 - 3 = -4$$

$$a_5 = a_{5-1} - 3 = a_4 - 3 = -4 - 3 = -7$$

$$a_6 = a_{6-1} - 3 = a_5 - 3 = -7 - 3 = -10$$

$$a_7 = a_{7-1} - 3 = a_6 - 3 = -10 - 3 = -13$$

$\therefore$ The next five terms are, $a_3 = -1, a_4 = -4, a_5 = -7, a_6 = -10, a_7 = -13$

(iii) $a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$

Given, first term (a_1) = -1

$$n^{\text{th}} \text{ term } (a_n) = \frac{a_n - 1}{n}, n \geq 2$$

To find next five terms i.e., a_2, a_3, a_4, a_5, a_6 we put $n = 2, 3, 4, 5, 6$ is an

$$a_2 = \frac{a_2 - 1}{2} = \frac{a_1}{2} = \frac{-1}{2}$$

$$a_3 = \frac{a_3 - 1}{3} = \frac{a_2}{3} = \frac{-1/2}{3} = \frac{-1}{6}$$

$$a_4 = \frac{a_4 - 1}{4} = \frac{a_3}{4} = \frac{-1/6}{4} = \frac{-1}{24}$$

$$a_5 = \frac{a_5 - 1}{5} = \frac{a_4}{5} = \frac{-1/24}{5} = \frac{-1}{120}$$

$\therefore$ The next five terms are,

$$a_2 = \frac{-1}{2}, a_3 = \frac{-1}{6}, a_4 = \frac{-1}{24}, a_5 = \frac{-1}{120}, a_6 = \frac{-1}{720}$$

(iv) $a_1 = 4, a_n = 4 a_{n-1} + 3, n > 1$

Given,

$$\text{First term } (a_1) = 4$$

$$n^{\text{th}} \text{ term } (a_n) = 4 a_{n-1} + 3, n > 1$$

To find next five terms i.e., a_2, a_3, a_4, a_5, a_6 we put $n = 2, 3, 4, 5, 6$ is a_n

Then, we get

$$a_2 = 4a_{2-1} + 3 = 4.a_1 + 3 = 4.4 + 3 = 19 (\because a_1 = 4)$$

$$a_3 = 4a_{3-1} + 3 = 4.a_2 + 3 = 4(19) + 3 = 79$$

$$a_4 = 4 a_{4-1} + 3 = 4.a_3 + 3 = 4(79) + 3 = 319$$

$$a_5 = 4 a_{5-1} + 3 = 4.a_4 + 3 = 4(319) + 3 = 1279$$

$$a_6 = 4.a_{6-1} + 3 = 4.a_5 + 3 = 4(1279) + 3 = 5119$$

$\therefore$ The required next five terms are,

$$a_2 = 19, a_3 = 79, a_4 = 319, a_5 = 1279, a_6 = 5119$$

Exercise – 9.2

1. For the following arithmetic progressions write the first term a and the common difference d :

(i) $-5, -1, 3, 7, \dots$

(ii) $\frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \frac{7}{5}, \dots$

(iii) $0.3, 0.55, 0.80, 1.05, \dots$

(iv) $-1.1, -3.1, -5.1, -7.1, \dots$

Sol:

We know that if a is the first term and d is the common difference, the arithmetic progression is $a, a + d, a + 2d, a + 3d, \dots$

(i) $-5, -1, 3, 7, \dots$

Given arithmetic series is

$$-5, -1, 3, 7, \dots$$

This is in the form of $a, a + d, a + 2d, a + 3d, \dots$ by comparing these two

$$a = -5, a + d = 1, a + 2d = 3, a + 3d = 7, \dots$$

$$\text{First term } (a) = -5$$

By subtracting second and first term, we get

$$(a + d) - (a) = d$$

$$-1 - (-5) = d$$

$$4 = d$$

$$\text{Common difference } (d) = 4.$$

$$(ii) \frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \frac{7}{5}, \dots$$

Given arithmetic series is,

$$\frac{1}{5}, \frac{3}{5}, \frac{5}{5}, \frac{7}{5}, \dots$$

This is in the form of $\frac{1}{5}, \frac{2}{5}, \frac{5}{5}, \frac{7}{5}, \dots$

$$a, a + d, a + 2d, a + 3d, \dots$$

By comparing this two, we get

$$a = \frac{1}{5}, a + d = \frac{3}{5}, a + 2d = \frac{5}{5}, a + 3d = \frac{7}{5}$$

$$\boxed{\text{First term } a = \frac{1}{5}}$$

By subtracting first term from second term, we get

$$d = (a + d) - (a)$$

$$d = \frac{3}{5} - \frac{1}{5}$$

$$d = \frac{2}{5}$$

$$\boxed{\text{common difference } (d) = \frac{2}{5}}$$

$$(iii) 0.3, 0.55, 0.80, 1.05, \dots$$

Given arithmetic series,

$$0.3, 0.55, 0.80, 1.05, \dots$$

General arithmetic series

$$a, a + d, a + 2d, a + 3d, \dots$$

By comparing,

$$a = 0.3, a + d = 0.55, a + 2d = 0.80, a + 3d = 1.05$$

First term $(a) = 0.3$.

By subtracting first term from second term. We get

$$d = (a + d) - (a)$$

$$d = 0.55 - 0.3$$

$$d = 0.25$$

Common difference $(d) = 0.25$

(iv) $-1.1, -3.1, -5.1, -7.1, \dots$

General series is

$$a, a + d, a + 2d, a + 3d, \dots$$

By comparing this two, we get

$$a = -1.1, a + d = -3.1, a + 2d = -5.1, a + 3d = -7.1$$

First term $(a) = -1.1$

Common difference $(d) = (a + d) - (a)$

$$= -3.1 - (-1.1)$$

Common difference $(d) = -2$

2. Write the arithmetic progressions write first term a and common difference d are as follows:

(i) $a = 4, d = -3$

(ii) $a = -1, d = \frac{1}{2}$

(iii) $a = -1.5, d = -0.5$

Sol:

We know that, if first term $(a) = a$ and common difference $= d$, then the arithmetic series is, $a, a + d, a + 2d, a + 3d, \dots$

(i) $a = 4, d = -3$

Given first term $(a) = 4$

Common difference $(d) = -3$

Then arithmetic progression is,

$$a, a + d, a + 2d, a + 3d, \dots$$

$$\Rightarrow 4, 4 - 3, a + 2(-3), 4 + 3(-3), \dots$$

$$\Rightarrow 4, 1, -2, -5, -8, \dots$$

(ii) $a = -1, d = \frac{1}{2}$

Given,

First term $(a) = -1$

Common difference $(d) = \frac{1}{2}$

Then arithmetic progression is,

$$\Rightarrow a, a + d, a + 2d, a + 3d, \dots$$

$$\Rightarrow -1, -1 + \frac{1}{2}, -1 + 2\frac{1}{2}, -1 + 3\frac{1}{2}, \dots$$

$$\Rightarrow -1, \frac{-1}{2}, 0, \frac{1}{2}, \dots$$

(iii) $a = -1.5, d = -0.5$

Given

First term $(a) = -1.5$

Common difference $(d) = -0.5$

Then arithmetic progression is

$$\Rightarrow a, a + d, a + 2d, a + 3d, \dots$$

$$\Rightarrow -1.5, -1.5 - 0.5, -1.5 + 2(-0.5), -1.5 + 3(-0.5)$$

$$\Rightarrow -1.5, -2, -2.5, -3, \dots$$

Then required progression is

$$-1.5, -2, -2.5, -3, \dots$$

3. In which of the following situations, the sequence of numbers formed will form an A.P.?

(i) The cost of digging a well for the first metre is Rs 150 and rises by Rs 20 for each succeeding metre.

(ii) The amount of air present in the cylinder when a vacuum pump removes each time $\frac{1}{4}$ of their remaining in the cylinder.

Sol:

(i) Given,

Cost of digging a well for the first meter $(c_1) = \text{Rs.}150$.

Cost rises by Rs.20 for each succeeding meter

Then,

Cost of digging for the second meter $(c_2) = \text{Rs.}150 + \text{Rs } 20$

$$= \text{Rs } 170$$

Cost of digging for the third meter $(c_3) = \text{Rs.}170 + \text{Rs } 20$

$$= \text{Rs } 210$$

Thus, costs of digging a well for different lengths are

$$150, 170, 190, 210, \dots$$

Clearly, this series is in $A \cdot p$.

With first term $(a) = 150$, common difference $(d) = 20$

(ii) Given

Let the initial volume of air in a cylinder be V liters each time $\frac{3}{4}$ th of air is remaining i.e.,

$$1 - \frac{1}{4}$$

First time, the air in cylinder is $\frac{3}{4}V$.

Second time, the air in cylinder is $\frac{3}{4}V$.

Third time, the air in cylinder is $\left(\frac{3}{4}\right)^2 V$.

Therefore, series is $V, \frac{3}{4}V, \left(\frac{3}{4}\right)^2 V, \left(\frac{3}{4}\right)^3 V, \dots$

4. Show that the sequence defined by $a_n = 5n - 7$ is an A.P., find its common difference.

Sol:

Given sequence is

$$a_n = 5n - 7$$

n^{th} term of given sequence $(a_n) = 5n - 7$

$(n+1)^{\text{th}}$ term of given sequence $(a_{n+1}) - a_n$

$$= (5n - 2) - (5n - 7)$$

$$= 5$$

$$\therefore d = 5$$

5. Show that the sequence defined by $a_n = 3n^2 - 5$ is not an A.P.

Sol:

Given sequence is,

$$a_n = 3n^2 - 5.$$

n^{th} term of given sequence $(a_n) = 3n^2 - 5.$

$(n+1)^{\text{th}}$ term of given sequence $(a_{n+1}) = 3(n+1)^2 - 5$

$$= 3(n^2 + 1^2 + 2n \cdot 1) - 5$$

$$= 3n^2 + 6n - 2$$

$\therefore$ The common difference $(d) = a_n + 1 - an$

$$d = (3n^2 + 6n - 2) - (3n^2 - 5)$$

$$= 3n^2 + 6n - 2 - 3n^2 + 5$$

$$= 6n + 3$$

Common difference (d) depends on 'n' value

$\therefore$ given sequence is not in A.P

6. The general term of a sequence is given by $a_n = -4n + 15$. Is the sequence an A.P.? If so, find its 15th term and the common difference.

Sol:

Given sequence is,

$$a_n = -4n + 15.$$

$$n^{th} \text{ term is } (a_n) = -4n + 15$$

$$(n+1)^{th} \text{ term is } (a_{n+1}) = -4(n+1) + 15$$

$$= -4n - 4 + 15$$

$$= -4n + 11$$

$$\text{Common difference } (d) = a_{n+1} - a_n$$

$$= (-4n + 11) - (-4n + 15)$$

$$= -4n + 11 + 4n - 15$$

$$d = -4$$

$$\text{Common difference } (d) = a_{n+1} - a_n$$

$$= (-4n + 11) + (-4n + 15)$$

$$= -4n + 11 + 4n - 15$$

$$d = -4.$$

Common difference (d) does not depend on 'n' value

$\therefore$ given sequence is in A.P

$$\Rightarrow 15^{th} \text{ term } a_{15} = -4(15) + 15$$

$$= -60 + 15$$

$$= -45$$

$$a_{15} = -45$$

7. Find the common difference and write the next four terms of each of the following arithmetic progressions:

(i) $1, -2, -5, -8, \dots$

(ii) $0, -3, -6, -9, \dots$

$$(iii) \quad -1, \frac{1}{4}, \frac{3}{2}, \dots$$

$$(iv) \quad -1, \frac{-5}{6}, \frac{-2}{3}, \dots$$

Sol:

$$(i) \quad 1, -2, -5, -8, \dots$$

Given arithmetic progression is,

$$a_1 = 1, a_2 = -2, a_3 = -5, a_4 = -8, \dots$$

$$\text{Common difference } (d) = a_2 - a_1$$

$$= -2 - 1$$

$$d = -3$$

To find next four terms

$$a_5 = a_4 + d = -8 - 3 = -11$$

$$a_6 = a_5 + d = -11 - 3 = -14$$

$$a_7 = a_6 + d = -14 - 3 = -17$$

$$a_8 = a_7 + d = -17 - 3 = -20$$

$$\therefore d = -3, a_5 = -11, a_6 = -14, a_7 = -17, a_8 = -20$$

$$(ii) \quad 0, -3, -6, -9, \dots$$

Given arithmetic progression is.

$$0, -3, -6, a_4 = -9, \dots$$

$$\text{Common difference } (d) = a_2 - a_1$$

$$= -3 - 0$$

$$d = -3$$

To find next four terms

$$a_5 = a_4 + d = -9 - 3 = -12$$

$$a_6 = a_5 + d = -12 - 3 = -15$$

$$a_7 = a_6 + d = -15 - 3 = -18$$

$$a_8 = a_7 + d = -18 - 3 = -21$$

$$\therefore d = -3, a_5 = -12, a_6 = -15, a_7 = -18, a_8 = -21$$

$$(iii) \quad -1, \frac{1}{4}, \frac{3}{2}, \dots$$

Given arithmetic progression is,

$$-1, \frac{1}{4}, \frac{3}{2}, \dots$$

$$a_1 = -1, a_2 = \frac{1}{4}, a_3 = \frac{3}{2}, \dots$$

$$\text{Common difference } (d) = a_2 - a_1$$

$$= \frac{1}{4} - (-1)$$

$$= \frac{1+4}{4}$$

$$d = \frac{5}{4}$$

To find next four terms,

$$a_4 = a_3 + d = \frac{3}{2} + \frac{5}{4} = \frac{6+5}{4} = \frac{11}{4}$$

$$a_5 = a_4 + d = \frac{11}{4} + \frac{5}{4} = \frac{16}{4}$$

$$a_6 = a_5 + d = \frac{16}{4} + \frac{5}{4} = \frac{21}{4}$$

$$a_7 = a_6 + d = \frac{21}{4} + \frac{5}{4} = \frac{26}{4}$$

$$\therefore d = \frac{5}{4}, a_4 = \frac{11}{4}, a_5 = \frac{16}{4}, a_6 = \frac{21}{4}, a_7 = \frac{26}{4}.$$

(iv) Given arithmetic progression is,

$$-1, \frac{-5}{6}, \frac{-2}{3}, \dots$$

$$a_1 = -1, a_2 = \frac{-5}{6}, a_3 = \frac{-2}{3}, \dots$$

$$\text{Common difference } (d) = a_2 - a_1$$

$$= \frac{-5}{6} - (-1)$$

$$= \frac{-5+6}{6}$$

$$= \frac{1}{6}$$

To find next four terms,

$$a_4 = a_3 + d = \frac{-2}{3} + \frac{1}{6} = \frac{-4+1}{6} = \frac{-3}{6} = -\frac{1}{2}.$$

$$a_5 = a_4 + d = \frac{-1}{2} + \frac{1}{6} = \frac{-3+1}{6} = \frac{-2}{6} = -\frac{1}{3}.$$

$$a_6 = a_5 + d = \frac{-1}{3} + \frac{1}{6} = \frac{-2+1}{6} = -\frac{1}{6}.$$

$$a_7 = a_6 + d = \frac{-1}{6} + \frac{1}{6} = 0.$$

$$\therefore d = \frac{1}{6}, a_4 = -\frac{1}{2}, a_5 = -\frac{1}{3}, a_6 = -\frac{1}{6}, a_7 = 0$$

8. Prove that no matter what the real numbers a and b are, the sequence with n th term $a + nb$ is always an A.P. What is the common difference?

Sol:

Given sequence $(a_n) = a + nb$

n^{th} term $(a_n) = a + nb$

$(n+1)^{\text{th}}$ term $(a_{n+1}) = a + (n+1)b$.

Common difference $(d) = a_{n+1} - a_n$

$$d = (a + (n+1)b) - (a + nb)$$

$$= \cancel{a} + \cancel{n}b + b - \cancel{a} - \cancel{n}b$$

$$= b$$

$\therefore$ common difference (d) does not depend on n^{th} value so, given sequence is in A.P. with $(d) = b$

9. Write the sequence with n th term :

$$(i) \quad a_n = 3 + 4n$$

Sol:

$$(i) \quad a_n = 3 + 4n$$

Given, n^{th} term $a_n = 3 + 4n$

$(n+1)^{\text{th}}$ term $a_{n+1} = 3 + 4(n+1)$

Common difference $(d) = a_{n+1} - a_n$

$$= (3 + 4(n+1)) - 3 - 4n$$

$$= 4.$$

$d = 4$ does not depend on n value so, the given series is in A.P. and the sequence is

$$a_1 = 3 + 4(1) = 3 + 4 = 7$$

$$a_2 = a_1 + d = 7 + 4 = 11; a_3 = a_2 + d = 11 + 4 = 15$$

$$\Rightarrow 7, 11, 15, 19, \dots$$

$$(ii) a_n = 5 + 2n$$

$$\text{Given, } n^{\text{th}} \text{ term } (a_n) = 5 + 2n$$

$$(n+1)^{\text{th}} \text{ term } (a_{n+1}) = 5 + 2(n+1)$$

$$= 7 + 2n$$

$$\text{Common difference } (d) = 7 + 2n - 5 - 2n$$

$$= 2.$$

$\therefore d = 2$ does not depend on n value given sequence is in A.p and the sequence is B_1

$$a_1 = 5 + 2 \cdot 1 = 7$$

$$a_2 = 7 + 2 = 9, a_3 = 9 + 2 = 11, a_4 = 11 + 2 = 13$$

$$\Rightarrow 7, 9, 11, 13, \dots$$

$$(iii) a_n = 6 - n$$

$$\text{Given, } n^{\text{th}} \text{ term } a_n = 6 - n$$

$$(n+1)^{\text{th}} \text{ term } a_{n+1} = 6 - (n+1)$$

$$= 5 - n$$

$$\text{Common difference } (d) = a_{n+1} - a_n$$

$$= (5 - n) - (6 - n)$$

$$= -1$$

$\therefore d = -1$ does not depend on n value given sequence is in A.p the sequence is

$$a_1 = 6 - 1 = 5, a_2 = 5 - 1 = 4, a_3 = 4 - 1 = 3, a_4 = 3 - 1 = 2$$

$$\Rightarrow 5, 4, 3, 2, 1, \dots$$

$$(iv) a_n = 9 - 5n$$

$$\text{Given, } n^{\text{th}} \text{ term } a_n = 9 - 5n$$

$$(n+1)^{\text{th}} \text{ term } a_{n+1} = 9 - 5(n+1)$$

$$= 4 - 5n$$

$$\text{Common difference } (d) = a_{n+1} - a_n$$

$$= (4 - 5n) - (9 - 5n)$$

$$= -5$$

$\therefore d = -5$ does not depend on n value given sequence is in A.p

The sequence is,

$$a_1 = 9 - 1.1 = 4$$

$$a_2 = 9 - 5.2 = -1$$

$$a_3 = 9 - 5.3 = -6$$

$$\Rightarrow 4, -1, -6, -11, \dots$$

10. Find out which of the following sequences are arithmetic progressions. For those which are arithmetic progressions, find out the common differences.

Sol:

- (i) 3, 6, 12, 24,

General arithmetic progression is $a, a + d, a + 2d, a + 3d, \dots$

Common difference (d) = Second term – first term

$$= (a + d) - a = d \text{ (or)}$$

$$= \text{Third term} - \text{second term}$$

$$= (a + 2d) - (a + d) = d$$

To check given sequence is in $A.p$ or not we use this condition.

Second term – First term = Third term – Second term

$$a_1 = 3, a_2 = 6, a_3 = 12, a_4 = 24$$

$$\text{Second term} - \text{First term} = 6 - 3 = 3$$

$$\text{Third term} - \text{Second term} = 12 - 6 = 6$$

This two are not equal so given sequence is not in $A.p$

- (ii) 0, -4, -8, -12,

In the given sequence

$$a_1 = 0, a_2 = -4, a_3 = -8, a_4 = -12$$

Check the condition

Second term – first term = third term – second term

$$a_2 - a_1 = a_3 - a_2$$

$$-4 - 0 = -8 - (-4)$$

$$-4 = -8 + 4$$

$$-4 = -4$$

Condition is satisfied $\therefore$ given sequence is in $A.p$ with common difference

$$(d) = a_2 - a_1 = -4$$

- (iii) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \dots$

In the given sequence

$$a_1 = \frac{1}{2}, a_2 = \frac{1}{4}, a_3 = \frac{1}{6}, a_4 = \frac{1}{8}$$

Check the condition

$$a_2 - a_1 = a_3 - a_2$$

$$\frac{1}{4} - \frac{1}{2} = \frac{1}{6} - \frac{1}{4}$$

$$\frac{1-2}{4} = \frac{4-6}{24}$$

$$\frac{-1}{4} = -\frac{2}{24}$$

$$\frac{-1}{4} \neq \frac{-1}{12}$$

Condition is not satisfied

$\therefore$ given sequence not in A.p

(iv) 12, 2, -8, -18,

In the given sequence

$$a_1 = 12, a_2 = 2, a_3 = -8, a_4 = -18$$

Check the condition

$$a_2 - a_1 = a_3 - a_2$$

$$2 - 12 = -8 - 2$$

$$-10 = -10$$

$\therefore$ given sequence is in A.p with common difference $d = -10$

(v) 3, 3, 3, 3,

In the given sequence

$$a_1 = 3, a_2 = 3, a_3 = 3, a_4 = 3$$

Check the condition

$$a_2 - a_1 = a_3 - a_2$$

$$3 - 3 = 3 - 3$$

$$0 = 0$$

$\therefore$ given sequence is in A.p with common difference $d = 0$

(vi) $p, p+90, p+80, p+270, \dots$ where $p = (999)$

In the given sequence

$$a_1 = p, a_2 = p+90, a_3 = p+180, a_4 = p+270$$

Check the condition

$$a_2 - a_1 = a_3 - a_2$$

$$\cancel{p} + 90 - \cancel{p} = \cancel{p} + 180 - \cancel{p} - 90$$

$$90 = 180 - 90$$

$$90 = 90$$

- (vii) 1.0, 1.7, 2.4, 3.1....,
 In the given sequence
 $a_1 = 1.0, a_2 = 1.7, a_3 = 2.4, a_4 = 3.1$
 Check the condition
 $a_2 - a_1 = a_3 - a_2$
 $1.7 - 1.0 = 2.4 - 1.7$
 $0.7 = 0.7$
 $\therefore$ The given sequence is in $A.p$ with $d = 0.7$
- (viii) -225, -425, -625, -825,.....
 In the given sequence
 $a_1 = 225, a_2 = -425, a_3 = -625, a_4 = -825$
 Check the condition
 $a_2 - a_1 = a_3 - a_2$
 $-425 + 225 = -625 + 425$
 $-200 = -200$
 $\therefore$ The given sequence is in $A.p$ with $d = -200$
- (ix) $10, 10 + 2^5, 10 + 2^6, 10 + 2^7, \dots$
 In the given sequence
 $a_1 = 10, a_2 = 10 + 2^5, a_3 = 10 + 2^6, a_4 = 10 + 2^7$
 Check the condition
 $a_2 - a_1 = a_3 - a_2$
 $10 + 2^5 - 10 = 10 + 2^6 - 10 - 2^5$
 $2^5 \neq 2^6 - 2^5$
 $\therefore$ The given sequence is not in $A.p$

Exercise – 9.3

1. Find:
 - (i) 10th term of the AP 1, 4, 7, 10....
 - (ii) 18th term of the AP $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \dots$
 - (iii) nth term of the AP 13, 8, 3, -2,.....
 - (iv) 10th term of the AP -40, -15, 10, 35,.....
 - (v) 8th term of the AP 11, 104, 91, 78,.....
 - (vi) 11th term of the AP 10.0, 10.5, 11.0, 11.2,.....
 - (vii) 9th term of the AP $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \dots$

Sol:

(i) Given A.p is

1, 4, 7, 10,

First term (a) = 1Common difference (d) = second term first term

$$= 4 - 1$$

$$= 3.$$

$$n^{\text{th}} \text{ term in an A.p} = a + (n-1)d$$

$$10^{\text{th}} \text{ term in an } 1 + (10-1)3$$

$$= 1 + 9 \cdot 3$$

$$= 1 + 27$$

$$= 28$$

(ii) Given A.p is

 $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \dots$ First term (a) = $\sqrt{2}$

Common difference = Second term – First term

$$= 3\sqrt{2} - \sqrt{2}$$

$$d = 2\sqrt{2}$$

$$n^{\text{th}} \text{ term in an A.p} = a + (n-1)d$$

$$18^{\text{th}} \text{ term of A.p} = \sqrt{2} + (18-1)2\sqrt{2}$$

$$= \sqrt{2} + 17 \cdot 2\sqrt{2}$$

$$= \sqrt{2}(1 + 34)$$

$$= 35\sqrt{2}$$

$$\therefore 18^{\text{th}} \text{ term of A.p is } 35\sqrt{2}$$

(iii) Given A.p is

13, 8, 3, -2,

First term (a) = 13Common difference (d) = Second term first term

$$= 8 - 13$$

$$= -5$$

$$n^{\text{th}} \text{ term of an A.p } a_n = a + (n-1)d$$

$$= 13 + (n-1) - 5$$

$$= 13 - 5n + 5$$

$$a_n = 18 - 5n$$

(iv) Given A.p is

$-40, -15, 10, 35, \dots$

First term $(a) = -40$

Common difference $(d) = \text{Second term} - \text{first term}$

$$= -15 - (-40)$$

$$= 40 - 15$$

$$= 25$$

n^{th} term of an A.p $a_n = a + (n-1)d$

10^{th} term of A.p $a_{10} = -40 + (10-1)25$

$$= -40 + 9.25$$

$$= -40 + 225$$

$$= 185$$

(v) Given sequence is

$117, 104, 91, 78, \dots$

First term $a = 117$

Common difference $(d) = \text{Second term} - \text{first term}$

$$= 104 - 117$$

$$= -13$$

n^{th} term $a_n = a + (n-1)d$

8^{th} term $a_8 = a + (8-1)d$

$$= 117 + 7(-13)$$

$$= 117 - 91$$

$$= 26$$

(vi) Given A.p is

$10.0, 10.5, 11.0, 11.5, \dots$

First term $(a) = 10.0$

Common difference $(d) = \text{Second term} - \text{first term}$

$$= 10.5 - 10.0$$

$$= 0.5$$

n^{th} term $a_n = a + (n-1)d$

11^{th} term $a_{11} = 10.0 + (11-1)0.5$

$$= 10.0 + 10 \times 0.5$$

$$= 10.0 + 5$$

$$=15.0$$

(vii) Given $A.p$ is

$$\frac{3}{4}, \frac{5}{4}, \frac{7}{4} + \frac{9}{4}, \dots$$

$$\text{First term } (a) = \frac{3}{4}$$

$$\text{Common difference } (d) = \text{Second term} - \text{first term}$$

$$= \frac{5}{4} - \frac{3}{4}$$

$$= \frac{2}{4}$$

$$n^{\text{th}} \text{ term } a_n = a + (n-1)d$$

$$9^{\text{th}} \text{ term } a_9 = a + (9-1)d$$

$$= \frac{3}{4} + 8 \cdot \frac{2}{4}$$

$$= \frac{3}{4} + \frac{16}{4}$$

$$= \frac{19}{4}$$

2. (i) Which term of the AP 3, 8, 13, is 248?
- (ii) Which term of the AP 84, 80, 76, ... is 0?
- (iii) Which term of the AP 4, 9, 14, is 254?
- (iv) Which term of the AP 21, 42, 63, 84, ... is 420?
- (v) Which term of the AP 121, 117, 113, ... is its first negative term?

Sol:

(i) Given $A.p$ is 3, 8, 13,

$$\text{First term } (a) = 3$$

$$\text{Common difference } (d) = \text{Second term} - \text{first term}$$

$$= 8 - 3$$

$$= 5$$

$$n^{\text{th}} \text{ term } (a_n) = a + (n-1)d$$

$$\text{Given } n^{\text{th}} \text{ term } a_n = 248$$

$$248 = 3 + (n-1) \cdot 5$$

$$248 = -2 + 5n$$

$$5n = 250$$

$$n = \frac{250}{5} = 50$$

50th term is 248.

(ii) Given A.p is 84, 80, 76,

First term (a) = 84

Common difference (d) = $a_2 - a$

$$= 80 - 84$$

$$= -4$$

n^{th} term (a_n) = $a + (n-1)d$

Given n^{th} term is 0

$$0 = 84 + (n-1) - 4$$

$$+84 = +4(n-1)$$

$$n-1 = \frac{\cancel{84}^{21}}{4} = 21$$

$$n = 21 + 1 = 22$$

22nd term is 0.

(iii) Given A.p 4, 9, 14,

First term (a) = 4

Common difference (d) = $a^2 - a$

$$= 9 - 4$$

$$= 5$$

n^{th} term (a_n) = $a + (n-1)d$

Given n^{th} term is 254

$$4 + (n-1)5 = 254$$

$$(n-1) \cdot 5 = 250$$

$$n-1 = \frac{250}{5} = 50$$

$$n = 51$$

$\therefore$ 51st term is 254.

(iv) Given A.p

21, 42, 63, 84,

$$a = 21, d = a_2 - a$$

$$= 42 - 21$$

$$= 21$$

$$n^{\text{th}} \text{ term } (a_n) = a + (n-1)d$$

$$\text{Given } n^{\text{th}} \text{ term} = 420$$

$$21 + (n-1)21 = 420$$

$$(n-1)21 = 399$$

$$n-1 = \frac{399}{21} = 19$$

$$n = 20$$

$$\therefore 20^{\text{th}} \text{ term is } 420.$$

(v) Given A.p is 121, 117, 113,

$$\text{First term } (a) = 121$$

$$\text{Common difference } (d) = 117 - 121$$

$$= -4$$

$$n^{\text{th}} \text{ term } (a) = a + (n-1)d$$

$$\text{Given } n^{\text{th}} \text{ term is negative i.e., } a_n < 0$$

$$121 + (n-1)(-4) < 0$$

$$121 + 4 - 4n < 0$$

$$125 - 4n < 0$$

$$4n > 125$$

$$n > \frac{125}{4}$$

$$n > 31.25$$

The integer which comes after 31.25 is 32.

$\therefore 32^{\text{nd}}$ term is first negative term

3. (i) Is 68 a term of the AP 7, 10, 13,?
 (ii) Is 302 a term of the AP 3, 8, 13,?
 (iii) Is -150 a term of the AP 11, 8, 5, 2, ...?

Sol:

In the given problem, we are given an A.p and the Value of one of its term

We need to find whether it is a term of the A.p or not so here we will use the formula

$$a_n = a + (n-1)d$$

(i) Here, A.p is 7, 10, 13,

$$a_n = 68, a = 7 \text{ and } d = 10 - 7 = 3$$

Using the above mentioned formula, we get

$$68 = 7 + (n-1)3$$

$$\Rightarrow 68 - 7 = 3n - 3$$

$$\Rightarrow 31 + 3 = 3n$$

$$\Rightarrow 64 = 3n$$

$$\Rightarrow n = \frac{64}{3}$$

Since, the value of n is a fraction.

Thus, 68 is not the term of the given A.p

(ii) Here, A.p is 3, 8, 13,

$$a_n = 302, a = 3$$

Common difference (d) = $8 - 3 = 5$ using the above mentioned formula, we get

$$302 = 3 + (n - 1)5$$

$$\Rightarrow 302 - 3 = 5n - 5$$

$$\Rightarrow 299 = 5n - 5$$

$$\Rightarrow 5n = 304$$

$$\Rightarrow n = \frac{304}{5}$$

Since, the value of ' n ' is a fraction. Thus, 302 is not the term of the given A.p

(iii) Here, A.p is 11, 8, 5, 2,

$$a_n = -150, a = 1 \text{ and } d = 8 - 11 = -3$$

Thus, using the above mentioned formula, we get

$$-150 = 11 + (n - 1)(-3)$$

$$\Rightarrow -150 - 11 = -3n + 3$$

$$\Rightarrow -161 - 3 = -3n$$

$$\Rightarrow -3n = -164$$

$$\Rightarrow n = \frac{164}{3}$$

Since, the value of n is a fraction. Thus, -150 is not the term of the given A.p

4. How many terms are there in the AP?

(i) 7, 10, 13, 43

(ii) $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, \dots, \frac{10}{3}$.

(iii) 7, 13, 19, 05

(iv) $18, 15\frac{1}{2}, 13, \dots, -47$

Sol:

(i) 7, 10, 13, 43

From given $A.p$

$$a = 7, d = 10 - 7 = 3, a_n = a + (n-1)d.$$

Let, $a_n = 43$ (last term)

$$7 + (n-1)3 = 43$$

$$(n-1) = \frac{26}{3} = 12$$

$$n = 13$$

$\therefore$ 13 terms are there in given $A.p$

$$(ii) \quad -1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, \dots, \frac{10}{3}.$$

From given $A.p$

$$a = -1, d = -\frac{5}{6} + 1, a_n = a + (n-1)d$$

$$= \frac{1}{6}$$

$$\text{Let, } a_n = \frac{10}{3} \text{ (last term)}$$

$$-1 + (n-1)\frac{1}{6} = \frac{10}{3}$$

$$(n-1) \times \frac{1}{6} = \frac{10}{3}$$

$$(n+1) = \frac{13 \times 6}{3} = 26$$

$$n = 27$$

$\therefore$ 27 terms are there in given $A.p$

$$(iii) \quad 7, 13, 19, \dots, 205$$

From the given $A.p$

$$a = 7, d = 13 - 7 = 6, a_n = a + (n-1)d$$

Let, $a_n = 205$ (last term)

$$7 + (n-1)6 = 205$$

$$(n-1) \cdot 6 = 198$$

$$n-1 = 33$$

$$n = 34$$

$\therefore$ 34 terms are there in given $A.p$

(iv) $18, 15\frac{1}{2}, 13, \dots - 47$

From the given A.p.,

$$a = 18, d = 15\frac{1}{2} - 18 = \frac{31}{2} - 18 = 15 \cdot 5 - 18 = -2 \cdot 5$$

$$a_n = a + (n-1) \cdot d$$

Let $a_n = -47$ (last term)

$$18 + (n-1) \cdot 2.5 = -47$$

$$12.5(n-1) = -65$$

$$n-1 = \frac{65}{2 \times 5} = \frac{65 \times 10}{25} = 26$$

$$n = 27$$

$\therefore$ 27 terms are there in given A.p

5. The first term of an AP is 5, the common difference is 3 and the last term is 80, find the number of terms.

Sol:

Given

First term (a) = 5

Common difference (d) = 3

Last term (l) = 80

To calculate no of terms in given A.p

$$a_n = a + (n-1)d$$

Let $a_n = 80$,

$$80 = 5 + (n-1) \cdot 3$$

$$75 = (n-1) \cdot 3$$

$$n-1 = \frac{75}{3} = 25$$

$$n = 26$$

$\therefore$ There are 26 terms.

6. The 6th and 17 terms of an A.P. are 19 and 41 respectively, find the 40th term.

Sol:

Given, $a_6 = 19, a_{17} = 41$

$$\Rightarrow a_6 = a + (6-1)d$$

$$19 = a + 5d \quad \dots\dots\dots(1)$$

$$\Rightarrow a_{17} = a + (17-1) \cdot d$$

$$41 = a + 16d \quad \dots\dots\dots(2)$$

Subtract (1) from (2)

$$a + 16d = 41$$

$$a + 5d = 19$$

$$0 + 11d = 22$$

$$d = \frac{22}{11} = 2$$

Substitute $d = 2$ in (1)

$$19 = a + 5(2)$$

$$9 = a$$

$$\therefore 40^{\text{th}} \text{ term } a_{40} = a + (40-1) \cdot d$$

$$= 9 + 39 \cdot 2$$

$$= 9 + 78$$

$$= 87$$

$$\therefore a_{40} = 87$$

7. If 9th term of an A.P. is zero, prove that its 29th term is double the 19th term.

Sol:

Given

$$9^{\text{th}} \text{ term of an A.P. } a_9 = 0, a_n = a + (n-1)d$$

$$a + (9-1) \cdot d = 0$$

$$a + 8d = 0$$

$$a = -8d$$

We have to prove

$$29^{\text{th}} \text{ term is double the } 19^{\text{th}} \text{ term } a_{29} = 2 \cdot a_{19}$$

$$a + (29-1)d = 2[a + (19-1)d]$$

$$a + 28d = 2[a + 18d]$$

$$\text{Put } a = -8d$$

$$-8d + 28d = 2[-8d + 18d]$$

$$20d = 2 \times 10d$$

$$20d = 20d$$

Hence proved

8. If 10 times the 10th term of an A.P. is equal to 15 times the 15th term, show that 25th term of the A.P. is zero.

Sol:

Given,

10 times of 10th term is equal to 15 times of 15th term.

$$10a_{10} = 15a_{15}$$

$$10[a + (10-1)d] = 15[a + (15-1)d] (\because a_n = a + (n-1)d)$$

$$10(a + 9d) = 15(a + 14d)$$

$$a + 9d = \frac{15}{10}(a + 14d)$$

$$a - \frac{3}{2}a = \frac{42d}{2} - 9d$$

$$-\frac{1}{2}a = \frac{24}{2} \cdot d$$

$$-a = +24 \cdot d$$

$$a = -24 \cdot d$$

We have to prove 25th term of A.P. is 0

$$a_{25} = 0$$

$$a + (25-1)d = 0$$

$$a + 24d = 0$$

$$\text{Put } a = -24d$$

$$-24 \times d + 24d = 0$$

$$0 = 0$$

Hence proved.

9. The 10th and 18th terms of an A.P. are 41 and 73 respectively. Find 26th term.

Sol:

Given,

$$a_{10} = 41, a_{18} = 73, a_n = a + (n-1) \cdot d$$

$$\Rightarrow a_{10} = a + (10-1) \cdot d$$

$$41 = a + 9d \quad \dots\dots\dots(1)$$

$$\Rightarrow a_{18} = a + (18-1)d$$

$$73 = a + 17d \quad \dots\dots\dots(2)$$

Subtract (1) from (2)

$$(2) - (1)$$

$$a + 17d = 73$$

$$a + 9d = 41$$

$$\hline 0 + 8d = 32$$

$$d = \frac{32}{8} = 4$$

Substitute $d = 4$ in (1)

$$a + 9 \cdot 4 = 41$$

$$a = 41 - 36$$

$$a = 5$$

$$26^{\text{th}} \text{ term } a_{26} = a + (26 - 1)d$$

$$= 5 + 25 \cdot 4$$

$$= 5 + 100$$

$$= 105$$

$$\therefore 26^{\text{th}} \text{ term } a_{26} = 105.$$

10. In a certain A.P. the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term.

Sol:

Given

24th term is twice the 10th term

$$a_{24} = 2 a_{10}$$

Let, first term of a square = a

Common difference = d

$$n^{\text{th}} \text{ term } a_n = a + (n - 1)d$$

$$a + (24 - 1)d = (a + (10 - 1)d) \cdot 2$$

$$a + 23d = 2(a + 9d)$$

$$(23 - 18)d = a$$

$$a = 5d$$

We have to prove

72nd term is twice the 34th term

$$a_{72} = 2a_{34}$$

$$a + (72 - 1)d = 2[a + (34 - 1)d]$$

$$a + 71d = 2a + 66d$$

Substitute $a = 5d$

$$5d + 71d = 2(5d) + 66d$$

$$76d = 10d + 66d$$

$$76d = 76d$$

Hence proved.

11. If $(m + 1)^{\text{th}}$ term of an A.P. is twice the $(n + 1)^{\text{th}}$ term, prove that $(3m + 1)^{\text{th}}$ term is twice the $(m+n+1)^{\text{th}}$ term.

Sol:

Given

$(m+1)^{\text{th}}$ term is twice the $(n+1)^{\text{th}}$ term.

First term = a

Common difference = d

n^{th} term $a_n = a + (n-1) \cdot d$

$$a_{m+1} = 2 a_n + 1$$

$$a + (m+1-1) \cdot d = 2(a + (n+1-1) \cdot d)$$

$$a + md = 2(a + nd)$$

$$a = (m - 2n)d$$

We have to prove

$(3m+1)^{\text{th}}$ term is twice the $(m+n+1)^{\text{th}}$ term

$$a_{3m+1} = 2 \cdot a_{m+n+1}$$

$$a + (3m+1-1) \cdot d = (a + (m+n+1-1) \cdot d)$$

$$a + 3m \cdot d = 2a + 2(m+n)d$$

Substitute $a = (m - 2n) \cdot d$

$$(m - 2n) \cancel{d} + 3m \cancel{d} = 2(m - 2n) \cancel{d} + 2(m+n) \cdot \cancel{d}$$

$$4m - 2n = 4m - 4n + 2n$$

$$4m - 2n = 4m - 2n$$

Hence proved.

12. If the n term of the A.P. 9, 7, 5, ... is same as the th term of the A.P. 15, 12, 9, ... find n .

Sol:

Given,

First sequence is 9, 7, 5,

$$a = 9, d = -2, a_n = a + (n-1)d$$

$$a_n = 9 + (n-1) \cdot (-2)$$

Second sequence is 15, 12, 9,

$$a = 15, d = 12 - 15 = -3, a_n = a + (n-1)d$$

$$a_n = 15 + (n-1) - 3$$

Given an. a_n are equal

$$9 - 2(n-1) = 15 - 3(n-1)$$

$$3(n-1) - 2(n-1) = 15 - 9$$

$$n - 1 = 6$$

$$n = 7$$

$\therefore 7^{\text{th}}$ term of two sequence are equal

13. Find the 12^{th} term from the end of the following arithmetic progressions:

(i) 3 5 7, 9, ... 201

(ii) 3, 8, 13, ... , 253

(iii) 1, 4, 7, 10, ..., 88

Sol:

(i) 3, 5, 7, 9, $2d$

First term $(a) = 3$

Common difference $(d) = 5 - 3 = 2$

12^{th} term from the end is can be considered as (1) last term = first term and common difference $= d^1 = -d$ n^{th} term from the end = last term $+(n-1) - d$

$$12^{\text{th}} \text{ term from end} = 201 + (12-1)(-2)$$

$$= 201 - 22$$

$$= 179$$

(ii) 3, 8, 13, 253

First term $= a = 3$

Common difference $d = 8 - 3 = 5$

Last term $(1) = 253$

n^{th} term of a sequence on $= a + (n-1) \cdot d$

To find n^{th} term from the end, we put last term (1) as ' a ' and common difference as $-d$

n^{th} term from the end = last term $+(n-1) \cdot -d$

$$12^{\text{th}} \text{ term from the end} = 253 + (12-1) - 5$$

$$= 253 - 55$$

$$= 198$$

$$\therefore 12^{\text{th}} \text{ term from the end} = 198$$

(iii) 1, 4, 7, 10, 88

First term $a = 1$

Common difference $d = 4 - 1 = 3$

Last term $(1) = 88$

$$n^{\text{th}} \text{ term } a_n = a + (n-1) \cdot d$$

$$n^{\text{th}} \text{ term from the end} = \text{last term} + (n-1) \cdot -d$$

$$12^{\text{th}} \text{ term from the end} = 88 + (12-1) \cdot -3$$

$$= 88 - 33$$

$$= 55$$

$$\therefore 12^{\text{th}} \text{ term from the end} = 55.$$

14. The 4th term of an A.P. is three times the first and the 7 term exceeds twice the third term by 1. Find the first term and the common difference.

Sol:

Given,

4^{th} term of an A.P. is three times the first term

$$a_4 = 3 \cdot a$$

$$n^{\text{th}} \text{ term of a sequence } a_n = a + (n-1) \cdot d$$

$$a + (4-1) \cdot d = 3a$$

$$a + 3d = 3a$$

$$3d = 2a$$

$$a = \frac{3}{2}d. \quad \dots\dots\dots(1)$$

Seventh term exceeds twice the third term by 1.

$$a_7 + 1 = 2 \cdot a_3$$

$$a + (7-1) \cdot d + 1 = 2(a + (3-1) \cdot d)$$

$$a + 6d + 1 = 2a + 4d$$

$$a = 2d + 1 \quad \dots\dots\dots(2)$$

By equating (1), (2)

$$\frac{3}{2}d = 2d + 1$$

$$\frac{3}{2}d - 2d = 1$$

$$\frac{3d - 4d}{2} = 1$$

$$-d = 2$$

$$d = -2$$

$$\text{Put } d = -2 \text{ in } a = \frac{3}{2}d$$

$$= \frac{3}{2} \cdot x$$

$$= -3$$

$\therefore$ First term $a = -3$, common difference $d = -2$.

15. Find the second term and n th term of an A.P. whose 6th term is 12 and the 8th term is 22.

Sol:

Given

$$a_6 = 12, a_8 = 22$$

$$n^{\text{th}} \text{ term of an A.P. } a_n = a + (n-1)d$$

$$a_6 = a + (n-1) \cdot d = a + (6-1)d = a + 5d = 12 \quad \dots\dots\dots(1)$$

Subtracting (1) from (2)

$$a + 7d = 22$$

$$(2) - (1) \Rightarrow \frac{a + 5d = 12}{0 + 2d = 10}$$

$$d = 5$$

$$a + 5d = 12$$

$$a + 5 \cdot 5 = 12$$

$$\text{Put } d = 5 \text{ in } a = 12 - 25$$

$$a = -13$$

$$\text{Second term } a_2 = a + (2-1) \cdot d$$

$$= a + d$$

$$= -13 + 5$$

$$a_1 = -8$$

$$n^{\text{th}} \text{ term } a_n = a + (n-1)d$$

$$= -13 + (n-1) \cdot 5$$

$$a_n = -18 + 5n$$

$$n^{\text{th}} \text{ term } a_n = a + (n-1)d$$

$$= -13 + (n-1) \cdot 5$$

$$a_n = -18 + 5n$$

$$\therefore a_2 = -8, a_n = -18 + 5n$$

16. How many numbers of two digits are divisible by 3?

Sol:

We observe that 12 is the first two-digit number divisible by 3 and 99 is the last two digit number divisible by 3. Thus, the sequence is

12, 15, 18, 99

This sequence is in A.P with

First term $(a) = 12$

Common difference $(d) = 15 - 12 = 3$

n^{th} term $a_n = 99$

n^{th} term of an A.P $(a_n) = a + (n - 1) \cdot d$

$$99 = 12 + (n - 1) \cdot 3$$

$$99 - 12 = n - 1 \cdot 3$$

$$\frac{87}{3} = n - 1$$

$$n = 30$$

$\therefore$ 30 term are there in the sequence

17. An A.P. consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32nd term.

Sol:

Given

No. of terms $= n = 60$

First term $(a) = 7$

Last term $a_{60} = 125$

$$a_{60} = a + (60 - 1) \cdot d \quad (\because a_n = a + (n - 1)d)$$

$$125 = 7 + 59 \cdot d$$

$$118 = 59d$$

$$d = \frac{118}{59} = 2$$

$$32^{\text{nd}} \text{ term } a_{32} = a + (32 - 1)d$$

$$= 7 + 31 \cdot 2$$

$$= 7 + 62$$

$$= 69$$

18. The sum of 4 and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 34. Find the first term and the common difference of the A.P.

Sol:

Given

$$a_4 + a_8 = 24$$

$$a_6 + a_{10} = 34$$

$$\Rightarrow a + (4-1)d + a + (8-1)d = 24$$

$$2a + 10d = 24$$

$$a + 5d = 12 \quad \dots\dots\dots(1)$$

$$\Rightarrow a_6 + a_{10} = 34$$

$$a + (6-1)d + a + (10-1)d = 34$$

$$2a + 14d = 34$$

$$a + 7d = 17 \quad \dots\dots\dots(2)$$

Subtract (1) from (2)

$$a + 7d = 17$$

$$\underline{a + 5d = 12}$$

$$2d = 5$$

$$d = \frac{5}{2}$$

$$\text{Put } d = \frac{5}{2} \text{ in } a + 5d = 12$$

$$a = 12 - 5 \cdot \frac{5}{2}$$

$$a = \frac{24 - 25}{2} = \frac{-1}{2}$$

$$\therefore a = -\frac{1}{2}, d = \frac{5}{2}$$

19. The first term of an A.P. is 5 and its 100th term is — 292. Find the 50th term of this A.P.

Sol:

Given,

$$a_{30} - a_{20} = a + (30-1)d - (a + (20-1)d) (\because a_n = a + (n-1)d)$$

$$= a + 29d - a - 19d$$

$$= 10d$$

$$(i) -9, -14, -19, -24, \dots\dots$$

Common difference (d) = second term – first term

$$= -14 - (-9)$$

$$= -14 + 9$$

$$d = 5$$

$$\text{Then } a_{30} - a_{20} = 10d$$

$$= 10.5$$

$$a_{30} - a_{20} = 50$$

$$\text{(ii) } a, a + d, a + 2d, a + 3d$$

$$\text{First term } (a) = a$$

$$\text{Common difference } (d) = d$$

$$a_{30} - a_{20} = a + (30 - 1)d - (a + (20 - 1)d)$$

$$= a + 29d - a - 19d$$

$$a_{30} - a_{20} = 10d$$

20. Find $a_{30} - a_{20}$ for the A.P.

$$\text{(i) } -9, -14, -19, -24, \dots$$

$$\text{(ii) } a, a + d, a + 2d, a + 3d, \dots$$

Sol:

Given,

$$a_{30} - a_{20} = a + (30 - 1)d - (a + (20 - 1)d) (\because a_n = a + (n - 1)d)$$

$$= a + 29d - a - 19d$$

$$= 10d$$

$$\text{(i) } -9, -14, -19, -24, \dots$$

Common difference (d) = second term – first term

$$= -14 - (-9)$$

$$= -14 + 9$$

$$d = 5$$

$$\text{Then } a_{30} - a_{20} = 10d$$

$$= 10.5$$

$$a_{30} - a_{20} = 50$$

$$\text{(ii) } a, a + d, a + 2d, a + 3d, \dots$$

$$\text{First term } (a) = a$$

$$a_{30} - a_{20} = a + (30 - 1)d - (a + (20 - 1)d)$$

$$= a + 29d - a - 19d$$

$$a_{30} - a_{20} = 10d$$

21. Write the expression $a_n - a_k$ for the A.P. $a, a + d, a + 2d, \dots$

Hence, find the common difference of the AP for which

$$\text{(i) } 11^{\text{th}} \text{ term } a_n = 5 \text{ and } 13^{\text{th}} \text{ term } a_{13} = 79$$

(ii) $a_{10} - a_5 = 200$

(iii) 20th term is 10 more than the 18th term.

Sol:

General arithmetic progression

$$a, a + d, a + 2d, \dots$$

$$a_b - a_k = a + (n-1)d - (a + (k-1)d) \quad (\because a_n = a + (n-1)d)$$

$$= a + (n-1)d - 2d + (k-1)d$$

$$a_n - a_k = (n-k)d \quad \dots\dots\dots(1)$$

(i) Given

$$11^{\text{th}} \text{ term } a_n = 5$$

$$13^{\text{th}} \text{ term } a_{13} = 79$$

By using (1) put $n = 13, k = 11$

$$a_n - a_k = (n-k) \cdot d$$

$$79 - 5 = (13 - 11) \cdot d$$

$$74 = 2 \times d$$

$$d = \frac{74}{2} = 37$$

(ii) Given

$$a_{10} - a_5 = 200$$

$$\text{From (1) } a_{10} - a_5 = (10 - 5)d$$

$$200 = 5 \cdot d$$

$$d = \frac{200}{5} = 40 \Rightarrow d = 40$$

(iii) Given

$$a_{20} - a_{18} = 10$$

$$a_{20} - a_{18} = 10$$

$$\text{By (1) } a_n - a_k = (n-k) \cdot d$$

$$a_{20} - a_{18} = (20 - 18) \cdot d$$

$$10 = 2 \cdot d$$

$$d = \frac{10}{2} = 5$$

$$\therefore d = 5$$

22. Find n if the given value of x is the n term of the given A.P.

(i) $1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \dots, x = \frac{141}{11}$

(ii) $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \dots : x = 550$

(iii) $-1, -3, -5, -7, \dots : x = -151$

(iv) $25, 50, 75, 100, \dots : c = 1000$

Sol:

(i) $25, 50, 75, 100, \dots : c = 1000$

First term $(a) = 25$

Common difference $(d) = 50 - 25 = 25$

n^{th} term $a_n = a + (n-1) \times d$

Given, $a_n = 1000$

$1000 = 25 + (n-1) \cdot 25$

$975 = (n-1) \times 25$

$n-1 = \frac{975}{25} = 39$

$n = 40$

(ii) Given sequence $-1, -3, -5, -7, \dots : x = -151$

First term $(a) = -1$

Common difference $(d) = -3 - (-1) = -3 + 1 = -2$

n^{th} term $a_n = a + (n-1)d$

Given $a_n = -151$,

$-151 = -1 + (n-1) - 2$

$-150 = 1(n-1) - 2$

$n-1 = \frac{150}{2} = 75$

$n = 76$

(iii) Given sequence is

$5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \dots : x = 550$

First term $(a) = 5\frac{1}{2} = \frac{11}{2}$

$= \frac{22-11}{2}$

$$= \frac{11}{2}$$

$$n^{\text{th}} \text{ term } a_n = a + (n-1)d$$

$$550 = \frac{11}{2} + (n-1) \cdot \frac{11}{2}$$

$$550 = \frac{11}{2} [1 + n - 1]$$

$$n = 550 \times \frac{2}{11}$$

$$n = 100$$

(iv) Given sequence is

$$1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \dots, x = \frac{141}{11}$$

First term $(a) = 1$

$$\text{Common difference } (d) = \frac{21}{11} - 1$$

$$= \frac{21-11}{11}$$

$$= \frac{10}{11}$$

$$n^{\text{th}} \text{ term } a_n = a + (n-1) \times d$$

$$\frac{171}{11} = 1 + (n-1) \cdot \frac{10}{11}$$

$$\frac{171}{11} - 1 = (n-1) \frac{10}{11}$$

$$\frac{171-11}{11} = (n-1) \frac{10}{11}$$

$$\frac{160}{11} = (n-1) \cdot \frac{10}{11}$$

$$n-1 = \frac{160}{11} \times \frac{11}{10}$$

$$n = 17$$

23. If an A.P. consists of n terms with first term a and n^{th} term 1 show that the sum of the m^{th} term from the beginning and the m^{th} term from the end is $(a + 1)$.

Sol:

First term of a sequence is a

Last term $= 1$

Total no. of terms = n

Common difference = d

m^{th} term from the beginning $a_m = a + (n-1) \cdot d$

m^{th} term from the end = last term $+ (n-1) - d$

$$a_n - m + 1 = 1 - (n-1) \times d$$

$$\Rightarrow a_m + a_n - m + 1 = a + (n-1)d + (1 - (n-1)d)$$

$$= a + (n-1)d + 1 - (n-1)d$$

$$a_m + a_n - m + 1 = a + 1$$

Hence proved

24. Find the arithmetic progression whose third term is 16 and seventh term exceeds its fifth term by 12.

Sol:

Given, $a_3 = 16$

$$a + (3-1)d = 16$$

$$a + 2d = 16. \quad \dots\dots\dots(1)$$

And $a_7 - 12 = a_5$

$$a + (7-1)d - 12 = a + (5-1)d \quad (\because a_n = a + (n-1)d)$$

$$\cancel{a} + 6d - 12 = \cancel{a} + 4d$$

$$2d = +12$$

$$d = +\frac{12}{2} = +6$$

Put $d = -6$ in (1)

$$a + 2(+6) = 16$$

$$a + 12 = 6$$

$$a = 284$$

Then the sequence is $a, a + d, a + 2d, a + 3d, \dots\dots$

$$\Rightarrow 28, 4, 10, 16, 22, \dots\dots$$

25. The 7th term of an A.P. is 32 and its 13th term is 62. Find the A.P.

Sol:

Given,

$$a + = 32$$

$$a + (7-1)d = 32$$

$$a + 6d = 32 \quad \dots\dots\dots(1)$$

And $a_{13} = 62$

$$a + (13-1)d = 62$$

$$a + 12d = 62 \quad \dots\dots\dots(2)$$

Subtract (1) from (2)

$$a + 12d = 62$$

$$(2) - (1) \Rightarrow \frac{a + 6d = 32}{0 + 6d = 32}$$

$$d = \frac{30}{6} = 5$$

Put $d = 5$ in $a + 6d = 32$

$$a + 6 \cdot 5 = 32$$

$$a = 2$$

Then the sequence is $a, a + d, a + 2d, a + 3d, \dots\dots\dots$

$$\Rightarrow 2, 7, 12, 17, \dots\dots\dots$$

26. Which term of the A.P. 3, 10, 17, ... will be 84 more than its 13th term?

Sol:

Given A.p is 3, 10, 17,

First term (a) = 3, Common difference (d) = $10 - 3$
 $= 7$

Let, n^{th} term of A.p will be 84 more than 13^{th} term

$$a_n = 84 + a_{13}$$

$$a + (n-1)d = a + (13-1)d + 84$$

$$(n-1)7 = 12 \cdot 7 + 84$$

$$(n-1) \cdot 7 = 168$$

$$n-1 = \frac{168}{7} = 24$$

$$n = 25$$

Hence 25^{th} term of given A.p is 84 more than 13^{th} term

27. Two arithmetic progressions have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Sol:

Let the two A.p is be $a_1, a_2, a_3, \dots\dots\dots$ and $b_1, b_2, b_3, \dots\dots\dots$

$$a_n = a_1 + (n-1)d \text{ and } b_n = b_1 + (n-1) \cdot d$$

Since common difference of two equations is same given difference between 100^{th} terms is 100

$$a_{100} - b_{100} = 100$$

$$a_1 + (99)d - b_1 - 99d = 100$$

$$a_1 - b_1 = 100 \quad \dots\dots\dots(1)$$

Difference between. 1000^{th} terms is

$$a_{1000} - b_{1000} = a_1 + (1000 - 1)d - (b_1 + (1000 - 1)d)$$

$$= a_1 + 999d - b_1 - 999d$$

$$= a_1 - b_1$$

$$= 100 \quad (\text{from (1)})$$

$\therefore$ Hence difference between 1000^{th} terms of two A.p is 100.

28. For what value of n, the nth terms of the arithmetic progressions 63, 65, 67, . . . and 3, 10, 17, . . . are equal?

Sol:

Given two A.p is are

63, 65, 67, and 3, 10,

First term of sequence 1 is $a_1 = 63$

Common difference $d_1 = 65 - 63$

$$= 2.$$

$$n^{th} \text{ term } (a_n) = a_1 + (n - 1)d$$

$$= 63 + (n - 1)2$$

First term of sequence 2 is $b_1 = 3$.

Common difference $d_2 = 10 - 3$

$$= 7$$

$$n^{th} \text{ term } (b_n) = b_1 + (n - 1)d_2$$

$$= 3 + (n - 1) \cdot 7$$

Let n^{th} terms of two sequence is equal

$$63 + (n - 1)2 = 3 + (n - 1) \times 7$$

$$60 = 5(n - 1)$$

$$n - 1 = \frac{60}{5} = 12$$

$$n = 13$$

$\therefore$ 13th term of both the sequence are equal.

29. How many multiples of 4 lie between 10 and 250?

Sol:

Multiple of 4 after 10 is 12 and multiple of 4 before 250 is $\frac{250}{4}$ remainder is 2, so,

$$250 - 2 = 248$$

248 is the last multiple of 4 before 250.

The sequence is

12,....., 248

With first term $(a) = 12$

Last term $(l) = 248$

Common difference $(d) = 4$

$$n^{\text{th}} \text{ term } a_n = a + (n-1) \cdot d$$

Here, n^{th} term $a_n = 248$

$$248 = 12 + (n-1) \times 4$$

$$236 = (n-1) \times 4$$

$$n-1 = \frac{236}{4} = 59$$

$$n = 60$$

$\therefore$ There are 60 terms between 10 and 250 which are multiples of 4

30. How many three digit numbers are divisible by 7?

Sol:

The three digit numbers are 100,.....999 105 is the first 3 digit number which is divisible by 7 when we divide 999 with 7 remainder is 5. So, $999 - 5 = 994$ is the last three digits divisible by 7 so, the sequence is

105,....., 994

First term $(a) = 105$

Last term $(l) = 994$

Common difference $(d) = -7$

Let there are n numbers in the sequence

$$a_n = 994$$

$$a + (n-1)d = 994$$

$$a + (n-1)d = 994$$

$$105 + (n-1)7 = 994$$

$$(n-1) \cdot 7 = 889$$

$$n-1 = \frac{889}{7} = 127$$

$$n = 128$$

$\therefore$ there are 128 numbers between 105, 994 which are divisible by 7

31. Which term of the arithmetic progression 8, 14, 20, 26, . . . will be 72 more than its 41st term?

Sol:

Given sequence

8, 14, 20, 26,

Let n^{th} term is 72 more than its 41st term

$$a_n = a_{41} + 72$$

For the given sequence

$$a = 8, d = 14 - 8 = 6$$

$$a + (n-1)d = 8 + (a+1)6 + 72$$

$$8 + (n-1)6 = 8 + (90) \cdot 6 + 72$$

$$(n-1)6 = 312$$

$$n-1 = \frac{312}{6} = 52$$

$$n = 53$$

$\therefore$ 53rd term is 72 more than 41st term

32. Find the term of the arithmetic progression 9, 12, 15, 18, . . . which is 39 more than its 36th term.

Sol:

Given A.p is 9, 12, 15,

For this $a = 9, d = 12 - 9 = 3$

Let n^{th} term is 39 more than its 36th term

$$a_n = 39 + a_{36}$$

$$a + (n-1)3 = 39 + a + (36-1) \cdot 3 \quad (\because a_n = a + (n-1)d)$$

$$(n-1)3 = 39 + 35 \cdot 3$$

$$(n-1) \times 3 = 144$$

$$n-1 = \frac{144}{3} = 48$$

$$n = 49$$

$\therefore 49^{\text{th}}$ term is 39 more than its 36^{th} term

33. Find the 8th term from the end of the A.P. 7, 10, 13, . . . , 184

Sol:

Given A.P. is 7, 10, 13, 184

$$a = 7, d = 10 - 7 = 3, l = 184$$

$$n^{\text{th}} \text{ term from the end} = l + (n-1)d$$

$$8^{\text{th}} \text{ term from the end} = 184 + (8-1) \times 3$$

$$= 184 - 21$$

$$= 163$$

$$\therefore 8^{\text{th}} \text{ term from the end} = 163$$

34. Find the 10th term from the end of the A.P. 8, 10, 12, . . . , 126.

Sol:

Given A.P. is 8, 10, 12, 126

$$a = 8, d = 10 - 8 = 2, l = 126$$

$$n^{\text{th}} \text{ term from the end} = l + (n-1)d$$

$$10^{\text{th}} \text{ term from the end} = 126 + (10-1) \times 2$$

$$= 126 - 18$$

$$= 108$$

$$\therefore 10^{\text{th}} \text{ term from the end} = 108$$

35. The sum of 4th and 8th terms of an A.P. is 24 and the sum of 6th and 10th terms is 44. Find the A.P.

Sol:

$$\text{Given, } a_4 + a_8 = 24$$

$$(a + (4-1)d) + (a + (8-1)d) = 24 \quad (\because a_n = a + (n-1)d)$$

$$2a + 10d = 24$$

$$a + 5d = 12 \quad \text{.....(1)}$$

$$\text{And } a_6 + a_{10} = 44$$

$$a + (6-1)d + a + (10-1)d = 44 \quad (\because a_n = a + (n-1)d)$$

$$2a + 14d = 44$$

$$a + 7d = 22 \quad \text{.....(2)}$$

Subtract (1) from (2)

$$a + 7d = 22$$

$$(2) - (1) \Rightarrow \frac{a + 5d = 12}{0 + 2d = 10}$$

$$d = 5$$

Put $d = 5$ in (1) $a + 5 \cdot 5 = 12$

$$a = -13$$

36. Which term of the A.P. 3, 15, 27, 39, . . . will be 120 more than its 21st term?

Sol:

Given A.p is

3, 15, 27, 39,

Let n^{th} term is 120 more than 21st term

$$\text{Then } a_n = 120 + a_{21}$$

For the given sequence

$$a = 3, d = 15 - 3 = 12$$

$$a + (n-1)d = 120 + a + (21-1)d$$

$$(n-1)12 = 120 + 20(12)$$

$$(n-1)12 = 360$$

$$(n-1) = \frac{360}{12} = 30$$

$$n = 31$$

$\therefore$ 31st term is 120 more than 21st term

37. The 17th term of an A.P. is 5 more than twice its 8th term. If the 11th term of the A.P. is 43, find the n^{th} term.

Sol:

Given

17th term of an A.p is 5 more than twice its 8th term

$$a_{17} = 5 + 2a_8$$

$$a + (17-1)d = 5 + 2(a + (8-1) \cdot d)$$

$$a + 16d = 5 + 2a + 14d$$

$$a + 5 = 2d \quad \dots\dots\dots(1)$$

And 11th term of the A.p is 43

$$a_{11} = 43$$

$$a + (11-1)d = 43$$

$$a_{11} = 43$$

$$a + (11-1)d = 43$$

$$a + 10d = 43 \quad \dots\dots\dots(2)$$

$$a + 10d = 43$$

$$(2) - (1) \Rightarrow \frac{a - 2d = +5}{a + 12d = 48}$$

$$d = \frac{48}{12} = 4$$

Put $d = 4$ in (1)

$$a + 5 = 2(4)$$

$$a = 3$$

$\therefore n^{\text{th}}$ term of given sequence is $a_n = a + (n-1)d$

$$= 3 + (n-1)4$$

$$= 3 + 4n - 4$$

$$= 4n - 1$$

$\therefore n^{\text{th}}$ term of given sequence $a_n = 4n - 1$

Exercise – 9.4

- The sum of three terms of an A.P. is 21 and the product of the first and the third terms exceeds the second term by 6, find three terms.

Sol:

Given,

Sum of three terms of on A.P is 21.

Product of first and the third term exceeds the second term by 6.

Let, the three numbers be $a-d$, a , $a+d$, with common difference d : then,

$$(a-d) + a + (a+d) = 21$$

$$3a = 21$$

$$a = \frac{21}{3} = 7$$

$$\text{and } (a-d)(a+d) = a + 6$$

$$a^2 - d^2 = a + 6$$

$$\text{Put } a = 7 \Rightarrow 7^2 - d^2 = 7 + 6$$

$$49 - 13 = d^2$$

$$d = \pm 6$$

$\therefore$ The three terms are $a-d$, a , $a+d$, i.e., 1, 7, 13.

2. Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers.

Sol:

Let, the three numbers are $a - d$, a , $a + d$.

Given,

$$(a - d) + a + (a + d) = 27$$

$$3a = 27$$

$$a = \frac{27}{3} = 9$$

$$\text{and, } (a - d)(a)(a + d) = 648$$

$$(a^2 - d^2)(a) = 648$$

Put $a = 9$, then

$$(9^2 - d^2) 9 = 648$$

$$81 - d^2 = \frac{648}{9} = 72$$

$$d^2 = 81 - 72$$

$$d^2 = 9$$

$$d = 3$$

$\therefore$ The three terms are $a - d$, a , $a + d$ i.e. 6, 9, 12.

3. Find the four numbers in A.P., whose sum is 50 and in which the greatest number is 4 times the least.

Sol:

Let, the four numbers be $a - 3d$, $a - d$, $a + d$, $a + 3d$, with common difference $2d$.

Given, sum is 50.

$$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 50$$

$$4a = 50$$

$$a = 12.5$$

greater number is 4 times the least

$$(a + 3d) = 4(a - 3d)$$

$$a + 3d = 4a - 12d$$

$$15d = 3a$$

Put $a = 12.5$

$$d = \frac{3}{15} \times 12.5$$

$$d = 2.5$$

$\therefore$ The four numbers are $a - 3d$, $a - d$, $a + d$, $a + 3d$ i.e., $12.5 - 3(2.5)$, $12.5 - 2.5$, $12.5 + 2.5$, $12.5 + 3(2.5)$

$$\Rightarrow 5, 10, 15, 20$$

4. The angles of a quadrilateral are in A.P. whose common difference is 10° . Find the angles.

Sol:

A quadrilateral has four angles. Given, four angles are in A.P with common difference 10.

Let, the four angles be, $a - 3d$, $a - d$, $a + d$, $a + 3d$ with common difference $= 2d$.

$$2d = 10$$

$$d = \frac{10}{2} = 5$$

In a quadrilateral, sum of all angles $= 360^\circ$

$$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 360$$

$$4a = 360$$

$$a = 360/4 = 90^\circ$$

$\therefore$ The angles are $a - 3d$, $a - d$, $a + d$, $a + 3d$ with $a = 90$, $d = 5$

i.e. $90 - 3(5)$, $90 - 5$, $90 + 3(5)$

$\Rightarrow 75^\circ, 85^\circ, 95^\circ, 105^\circ$.

5. The sum of three numbers in A.P. is 12 and the sum of their cubes is 288. Find the numbers.

Sol:

2, 4, 6, or 6, 4, 2.

6. Find the value of x for which $(8x + 4)$, $(6x - 2)$ and $(2x + 7)$ are in A.P.

Sol:

Given,

$8x + 4$, $6x - 2$, $2x + 7$ are in A.P.

If the numbers a , b , c are in A.P. then condition is $2b = a + c$.

Then, $2(6x - 2) = 8x + 4 + 2x + 7$

$$12x - 4 = 10 + 11$$

$$2x = 15$$

$$x = \frac{15}{2}$$

7. If $x + 1$, $3x$ and $4x + 2$ are in A.P., find the value of x .

Sol:

Given numbers

$x + 1$, $3x$, $4x + 2$ are in AP

If a , b , c are in AP then $2b = a + c$

Then $2(3x) = x + 1 + 4x + 2$

$$6x = 5x + 3$$

$$x = 3$$

8. Show that $(a - b)^2, (a^2 + b^2)$ and $(a + b)^2$ are in A.P.

Sol:

We have to show, $(a - b)^2, (a^2 + b^2)$ and $(a + b)^2$ are in AP.

If they are in AP. Then they have to satisfy the condition

$$2b = a + c$$

$$2(a^2 + b^2) = (a - b)^2 + (a + b)^2$$

$$2a^2 + 2b^2 = a^2 + 2ab + b^2 + a^2 + 2ab + b^2$$

$$2a^2 + 2b^2 = 2a^2 + 2b^2.$$

They satisfy the condition means they are in AP.

Exercise – 9.5

1. Find the sum of the following arithmetic progressions:

(i) 50, 46, 42, ... to 10 terms

(ii) 1, 3, 5, 7, ... to 12 terms

(iii) 3, $9/2$, 6, $15/2$, ... to 25 terms

(iv) 41, 36, 31, ... to 12 terms

(v) $a + b, a - b, a - 3b, \dots$ to 22 terms

(vi) $(x - y)^2, (x^2 + y^2), (x + y)^2, \dots$, to n terms

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \dots$ to n terms

(viii) $-26, -24, -22, \dots$ to 36 terms

Sol:

In an A.P let first term = a , common difference = d , and there are n terms. Then, sum of n terms is,

$$S_n = \frac{n}{2} \{2a + (n - 1)d\}$$

(i) Given progression is,

50, 46, 42,to 10 term.

First term (a) = 50

Common difference (d) = $46 - 50 = -4$

n^{th} term = 10

$$\text{Then } S_{10} = \frac{10}{2} \{2 \cdot 50 + (10 - 1) \cdot (-4)\}$$

$$= 5 \{100 - 36\}$$

$$= 5 \{64\}$$

$$= 5 \times 64$$

$$\therefore S_{10} = 320$$

(ii) Given progression is,

1, 3, 5, 7,to 12 terms

First term difference (d) = $3 - 1 = 2$

n^{th} term = 12

$$\begin{aligned}
 \text{Sum of } n^{\text{th}} \text{ terms } S_{12} &= \frac{12}{2} \times \{2.1 + (12 - 1).2\} \\
 &= 6 \times \{2 + 22\} = 6.24 \\
 \therefore S_{12} &= 144.
 \end{aligned}$$

(iii) Given expression is

$$3, \frac{9}{2}, 6, \frac{15}{2}, \dots \dots \text{to } 25 \text{ terms}$$

First term (a) = 3

$$\text{Common difference (d)} = \frac{9}{2} - 3 = \frac{3}{2}$$

Sum of n^{th} terms S_n , given $n = 25$

$$S_{25} = \frac{n}{2} (2a + (n - 1).d)$$

$$S_{25} = \frac{25}{2} \left(2.3 + (25 - 1). \frac{3}{2} \right)$$

$$= \frac{25}{2} \left(6 + 24. \frac{3}{2} \right)$$

$$= \frac{25}{2} (6 + 36)$$

$$= \frac{25}{2} (42)$$

$$\therefore S_{25} = 525$$

(iv) Given expression is,

$$41, 36, 31, \dots \dots \text{to } 12 \text{ terms.}$$

First term (a) = 41

$$\text{Common difference (d)} = 36 - 41 = -5$$

Sum of n^{th} terms S_n , given $n = 12$

$$S_{12} = \frac{n}{2} (2a + (n - 1).d)$$

$$= \frac{12}{2} (2.41 + (12 - 1). -5)$$

$$= 6(82 + 11. (-5))$$

$$= 6(27)$$

$$= 162$$

$$\therefore S_{12} = 162.$$

(v) $a + b, a - b, a - 3b, \dots \dots \text{to } 22 \text{ terms}$

First term (a) = $a + b$

$$\text{Common difference (d)} = a - b - a - b = -2b$$

$$\text{Sum of } n^{\text{th}} \text{ terms } S_n = \frac{n}{2} \{2a + (n - 1).d\}$$

Here $n = 22$

$$S_{22} = \frac{22}{2} \{2.(a + b) + (22 - 1). -2b\}$$

$$= 11 \{2(a + b) - 22b\}$$

$$= 11 \{2a - 20b\}$$

$$= 22a - 440b$$

$$\therefore S_{22} = 22a - 440b$$

- (vi)
- $(x - y)^2, (x^2 + y^2), (x + y)^2, \dots$
- to
- n
- terms

First term (a) = $(x - y)^2$

Common difference (d) = $x^2 + y^2 - (x - y)^2$

$$= x^2 + y^2 - (x^2 + y^2 - 2xy)$$

$$= x^2 + y^2 - x^2 - y^2 + 2xy$$

$$= 2xy$$

Sum of n^{th} terms $S_n = \frac{n}{2} \{2a(n - 1).d\}$

$$= \frac{n}{2} \{2(x - y)^2 + (n - 1).2xy\}$$

$$= n\{(x - y)^2 + (n - 1)xy\}$$

$$\therefore S_n = n\{(x - y)^2 + (n - 1).xy\}$$

- (vii)
- $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \dots$
- to
- n
- terms

First term (a) = $\frac{x-y}{x+y}$

Common difference (d) =
$$\begin{aligned} &= \frac{3x-2y}{x+y} - \frac{x-y}{x+y} \\ &= \frac{3x-2y-x+y}{x+y} \\ &= \frac{2x-y}{x+y} \end{aligned}$$

Sum of n terms $S_n = \frac{n}{2} \{2a + (n - 1).d\}$

$$= \frac{n}{2} \left\{ 2 \cdot \frac{x-y}{x+y} + (n - 1) \cdot \frac{2x-y}{x+y} \right\}$$

$$= \frac{n}{2(x+y)} \{2(x - y) + (n - 1)(2x - y)\}$$

$$= \frac{n}{2(x+y)} \{2x - 2y + 2nx - ny - 2x + y\}$$

$$= \frac{n}{2(x+y)} \{n(2x - y) - y\}$$

$$\therefore S_n = \frac{n}{2(x+y)} \{n(2x - y) - y\}$$

- (viii) Given expression
- $-26, -24, -22, \dots$
- To 36 terms

First term (a) = -26

Common difference (d) = $-24 - (-26) = -24 + 26 = 2$

Sum of n terms $S_n = \frac{n}{2} \{2a + (n - 1)d\}$

Sum of n terms $S_n = \frac{36}{2} \{2 \cdot -26 + (36 - 1)2\}$

$$= 18[-52 + 70]$$

$$= 18 \cdot 18$$

$$= 324$$

$$\therefore S_n = 324$$

2. Find the sum to
- n
- term of the A.P.
- $5, 2, -1, -4, -7, \dots$

Sol:Given AP is $5, 2, -1, -4, -7, \dots$

$$a = 5, d = 2 - 5 = -3$$

$$\begin{aligned} S_n &= \frac{n}{2} \{2a + (n - 1)d\} \\ &= \frac{n}{2} \{2 \cdot 5 + (n - 1) - 3\} \\ &= \frac{n}{2} \{10 - 3(n - 1)\} \\ &= \frac{n}{2} \{13 - 3n\} \\ \therefore S_n &= \frac{n}{2} (13 - 3n) \end{aligned}$$

3. Find the sum of n terms of an A.P. whose n th term is given by $a_n = 5 - 6n$.

Sol:

$$\text{Given } n\text{th term } a_n = 5 - 6n$$

$$\text{Put } n = 1, a_1 = 5 - 6 \cdot 1 = -1$$

$$\text{We know, first term } (a_1) = -1$$

$$\text{Last term } (a_n) = 5 - 6n = 1$$

$$\begin{aligned} \text{Then } S_n &= \frac{n}{2} (-1 + 5 - 6n) \\ &= \frac{n}{2} (4 - 6n) = \frac{n}{2} (2 - 3n) \end{aligned}$$

4. If the sum of a certain number of terms starting from first term of an A.P. is 25, 22, 19, ... is 116. Find the last term.

Sol:

$$\text{Given AP is } 25, 22, 19, \dots$$

$$\text{First term } (a) = 25, d = 22 - 25 = -3.$$

$$\text{Given, } S_n = \frac{n}{2} (2a + (n - 1)d)$$

$$116 = \frac{n}{2} (2 \times 25 + (n - 1) - 3)$$

$$232 = n(50 - 3(n - 1))$$

$$232 = n(53 - 3n)$$

$$232 = 53n - 3n^2$$

$$3n^2 - 53n + 232 = 0$$

$$(3n - 29)(n - 8) = 0$$

$$\therefore n = 8$$

$$\Rightarrow a_8 = 25 + (8 - 1) - 3$$

$$\therefore n = 8, a_8 = 4$$

$$= 25 - 21 = 4$$

5. (i) How many terms of the sequence 18, 16, 14, ... should be taken so that their
(ii) How many terms are there in the A.P. whose first and fifth terms are -14 and 2 respectively and the sum of the terms is 40 ?
(iii) How many terms of the A.P. 9, 17, 25, ... must be taken so that their sum is 636 ?

(iv) How many terms of the A.P. 63, 60, 57, . . . must be taken so that their sum is 693?

Sol:

(i) Given sequence, 18, 16, 14, ...

$$a = 18, d = 16 - 18 = -2.$$

Let, sum of n terms in the sequence is zero

$$S_n = 0$$

$$\frac{n}{2}(2a + (n - 1)d) = 0$$

$$\frac{n}{2}(2 \cdot 18 + (n - 1) - 2) = 0$$

$$n(18 - (n - 1)) = 0$$

$$n(19 - n) = 0$$

$$n = 0 \text{ or } n = 19$$

(ii) $\because n = 0$ is not possible. Therefore, sum of 19 numbers in the sequence is zero.

$$\text{Given, } a = -14, a_5 = 2$$

$$a + (5 - 1)d = 2$$

$$-14 + 4d = 2$$

$$4d = 16 \Rightarrow d = 4$$

Sequence is $-14, -10, -6, -2, 2, \dots$

$$\text{Given } S_n = 40$$

$$40 = \frac{n}{2}\{2(-14) + (n - 1)4\}$$

$$80 = n(-28 + 4n - 4)$$

$$80 = n(-32 + 4n)$$

$$4(20) = 4n(-8 + n)$$

$$n^2 - 8n - 20 = 0$$

$$(n - 10)(n + 2) = 0$$

$$n = 10 \text{ or } n = -2$$

$\therefore$ Sum of 10 numbers is 40 (Since -2 is not a natural number)

(iii) Given AP 9, 17, 25,

$$a = 9, d = 17 - 9 = 8, \text{ and } S_n = 636$$

$$636 = \frac{n}{2}(2 \cdot 9 + (n - 1)8)$$

$$1272 = n(18 - 8 + 8n)$$

$$1272 = n(10 + 8n)$$

$$2 \times 636 = 2n(5 + 4n)$$

$$636 = 5n + 4n^2$$

$$4n^2 + 5n - 636 = 0$$

$$(4n + 53)(n - 12) = 0$$

$\therefore n = 12$ (Since $n \frac{-53}{4}$ is not a natural number)

Therefore, value of n is 12.

(iv) Given AP, 63, 60, 57,

$$a = 63, d = 60 - 63 = -3 \quad S_n = 693$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$693 = \frac{n}{2}(2 \cdot 63 + (n-1) \cdot (-3))$$

$$1386 = n(126 - 3n + 3)$$

$$1386 = (129 - 3n)n$$

$$3n^2 - 129n + 1386 = 0$$

$$n^2 - 43n + 462 = 0$$

$$n = 21, 22$$

$\therefore$ Sum of 21 or 22 term is 693

6. The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Sol:

Given, $a = 17, l = 350, d = 9$

$$l = a_n = a + (n-1)d$$

$$350 = 17 + (n-1)9$$

$$333 = (n-1)9$$

$$n-1 = \frac{333}{9} = 37$$

$$n = 38$$

$\therefore$ 38 terms are there

$$S_n = \frac{n}{2}\{a + l\}$$

$$= \frac{38}{2}\{17 + 350\}$$

$$= 19.367$$

$$\therefore S_n = 6973$$

7. The third term of an A.P. is 7 and the seventh term exceeds three times the third term by 2. Find the first term, the common difference and the sum of first 20 terms.

Sol:

Given, $a_3 = 7$ and $3a_3 + 2 = a_7$

$$a_7 = 3 \cdot 7 + 2$$

$$a_7 = 21 + 2 = 23$$

$$\therefore a_n = a + (n-1)d$$

$$a_3 = a + (3-1)d \text{ and } a_7 = a + (7-1)d$$

$$7 = a + 2d \dots (i) \quad 23 = a + 6d \dots (ii)$$

Subtract (i) from (ii)

$$(ii) - (i) \Rightarrow \quad a + 6d = 23$$

$$\underline{a + 2d = 7}$$

$$4d = 16$$

$$d = 4$$

$$\text{Put } d = 4 \text{ in (i)} \Rightarrow 7 = a + 2 \cdot 4$$

$$a = 7 - 8 = -1$$

Given to find sum of first 20 terms.

$$S_{20} = \frac{20}{2} \{-2 + (10 - 1)4\}$$

$$= 10(-2 + 76)$$

$$\therefore S_{20} = 740$$

8. The first term of an A.P. is 2 and the last term is 50. The sum of all these terms is 442. Find the common difference.

Sol:

$$\text{Given } a = 2, l = 50, S_n = 442$$

$$S_n = \frac{n}{2}(a + l)$$

$$442 = \frac{n}{2}(2 + 50)$$

$$442 = \frac{n}{2} \cdot 52$$

$$\therefore n = \frac{442 \cdot 2}{52} = 17$$

$$\text{Given, } a_n = l = 50$$

$$50 = 2 + (17 - 1)d$$

$$48 = 16 \times d$$

$$d = \frac{48}{16} = 3$$

$$\therefore d = 3$$

9. If 12th term of an A.P. is -13 and the sum of the first four terms is 24, what is the sum of first 10 terms?

Sol:

$$\text{Given, } a_{12} = -13, a + a_2 + a_3 + a_4 = 24$$

$$S_4 = \frac{4}{2}(2a + 3d) = 24$$

$$2a + 3d = \frac{24}{2} = 12 \dots (i)$$

$$\Rightarrow a + (12 - 1)d = -13$$

$$a + 11d = -13 \dots (ii)$$

Subtract (i) from (ii) $\times 2$

$$2 \times (ii) - (i) \Rightarrow 2a + 22d = -28$$

$$\underline{2a + 3d = 12}$$

$$19d = -38$$

$$d = \frac{-38}{19} = -2$$

$$\text{put } d = -2 \text{ in (ii)}$$

$$a + 11(-2) = -13$$

$$a = -13 + 22$$

$$a = 9$$

Given to find sum of first 10 terms.

$$\begin{aligned} S_{10} &= \frac{10}{2} \{2(a) + (10 - 1) - 2\} \\ &= 5(18 - 18) \\ &= 0 \end{aligned}$$

$$\therefore S_{10} = 0$$

10. Find the sum of first 22 terms of an A.P. in which $d = 22$ and $a = 149$.

Sol:

$$\text{Given, } d = 22, a_{22} = 149$$

$$a + (22 - 1)d = 149$$

$$a = -313$$

$$\text{Given, to find } S_{22} = \frac{22}{2} [2a + (22 - 1)d]$$

$$= 11[2(-313) + 21 \cdot 22]$$

$$= 11[-626 + 462]$$

$$= 11 - 164$$

$$= -1804$$

$$\therefore S_{22} = -1804$$

11. Find the sum of all natural numbers between 1 and 100, which are divisible by 3.

Sol:

The numbers between 1 and 100 which are divisible by 3 are 3, 6, 9, ..., 99.

In this sequence, $a = 3$, $d = 3$, $a_n = 99$

$$99 = a + (n - 1)d$$

$$99 = 3 + (n - 1)3$$

$$99 = 3[1 + n - 1]$$

$$n = \frac{99}{3} = 33$$

$\therefore$ There are 33 numbers in the given sequence

$$S_{33} = \frac{33}{2} (2 \cdot 3 + (33 - 1)3) \left(\because S_n = \frac{n}{2} (2a + (n - 1)d) \right)$$

$$= \frac{33}{2} (6 + 96)$$

$$= \frac{33}{2} \times 102$$

$$= 1683$$

$\therefore$ Sum of all natural numbers between 1 and 100, which are divisible by 3 is 1683.

12. Find the sum of first n odd natural numbers.

Sol:

The sequence is, 1, 3, 5, n .

In this first term (a) = 1, common difference (d) = 2

$$\begin{aligned} S_n &= \frac{n}{2}(2a + (n - 1)d) \\ &= \frac{n}{2}(2.1 + (n - 1)2) \\ &= \frac{n}{2} \times 2(1 + n - 1) \\ &= n^2. \end{aligned}$$

$\therefore$ Sum of first n odd natural numbers is n^2 .

13. Find the sum of all odd numbers between (i) 0 and 50 (ii) 100 and 200.

Sol:

- (i) Odd numbers between 0 and 50 are 1, 3, 5,, 49

In this $a = 1$, $d = 2$, $l = 49 = a_n$

$$49 = 1 + (n - 1)2 \quad (\because a_n = a + (n - 1)d)$$

$$48 = (n - 1)2$$

$$n - 1 = \frac{48}{2} = 24$$

$$n = 25.$$

$\therefore$ There are 25 terms

$$\begin{aligned} S_{25} &= \frac{25}{2}(1 + 49) \quad \left(\because S_n = \frac{n}{2}(a + l) \right) \\ &= \frac{25}{2} \times 50 = 625 \end{aligned}$$

$\therefore$ Sum of all odd numbers between 0 and 50 is 625.

- (ii) Odd numbers between 100 and 200 are 101, 103, 199

In this $a = 101$, $d = 2$, $l = a_n = 199$

$$199 = 101 + (n - 1)2$$

$$n - 1 = \frac{98}{2} = 49$$

$$n = 50$$

$\therefore$ There are 50 terms.

$$\begin{aligned} S_{50} &= \frac{50}{2}(101 + 199) \quad \left(\because S_n = \frac{n}{2}(a + l) \right) \\ &= \frac{50}{2} \times 300 \\ &= 7500 \end{aligned}$$

$\therefore$ Sum of all odd numbers between 100 and 200 is 7500.

14. Show that the sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667.

Sol:

Odd integers between 1 and 1000 which are divisible by 3 are 3, 6, 9, 15 999.

In this $a = 3$, $d = 3$, $l = a_n = 999$

$$999 = 3 + (n - 1)3 \quad (\because a_n = a + (n - 1)d)$$

$$999 = 3[1 + (n - 1)]$$

$$\therefore 2n - 1 = \frac{999}{3} = 333 \Rightarrow n = \frac{334}{2} = 167$$

$\therefore$ There are 167 numbers.

$$S_{167} = \frac{167}{2} [3 + 999]$$

$$= \frac{167}{2} \times 1002 = 83667$$

$$\therefore S_{167} = 83667$$

$\therefore$ Sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667.

15. Find the sum of all integers between 84 and 719, which are multiples of 5.

Sol:

The numbers between 84 and 719, which are multiples of 5 are 85, 90, 95, 715.

In this, $a = 85$, $d = 5$, $a_n = l = 715$

$$715 = 85 + (n - 1)5 \quad (\because a_n = a + (n - 1)d)$$

$$630 = (n - 1)5$$

$$n - 1 = 126$$

$$n = 127$$

$$\therefore S_n = \frac{127}{2} (85 + 715) \quad \left(\because S_n = \frac{n}{2} (a + l) \right)$$

$$= \frac{127}{2} \times 800 = 50800$$

$\therefore$ Sum of all integers between 84 and 719, which are multiples of 5 is 50800.

16. Find the sum of all integers between 50 and 500, which are divisible by 7.

Sol:

Numbers between 50 and 500, which are divisible by 7 are 56, 63, 497.

In this $a = 56$, $d = 7$, $l = a_n = 497$

$$497 = 56 + (n - 1)7$$

$$441 = (n - 1)7$$

$$n - 1 = \frac{441}{7} = 63$$

$$n = 64$$

$\therefore$ There are 64 terms.

$$S_{64} = \frac{64}{2} (56 + 497)$$

$$= 32 \times 553 = 17696$$

$\therefore$ Sum of all integers between 50 and 500, which are divisible by 7 is 17696.

17. Find the sum of all even integers between 101 and 999.

Sol:

Even integers between 101 and 999 are 102, 104,998

$$a = 102, d = 2, a_n = l = 998$$

$$998 = 102 + (n - 1) \times 2 \quad (\because a_n = a + (n - 1)d)$$

$$896 = (n - 1)(2)$$

$$n - 1 = 448$$

$$n = 449.$$

$\therefore$ 449 terms are there

$$S_{449} = \frac{449}{2} [102 + 998]$$

$$= \frac{449}{2} \times 1100 = 246950$$

$\therefore$ Sum of all even integers between 101 and 999 is 24690

18. Find the sum of all integers between 100 and 550, which are divisible by 9.

Sol:

Integers between 100 and 550 which are divisible by 9 are 108, 117,, 549.

In this $a = 108, d = 9, a_n = l = 549$

$$549 = 108 + (n - 1) \times 9 \quad (\because a_n = a + (n - 1)d)$$

$$441 = (n - 1) \times 9$$

$$n - 1 = \frac{441}{9} = 49$$

$$n = 50.$$

$$\therefore S_{50} = \frac{50}{2} \{108 + 549\} \quad \left(\because S_n = \frac{n}{2} (10 + 1) \right)$$

$$= 25 \times 657$$

$$= 16425$$

$\therefore$ Sum of all integers between 100 and 550, which are divisible by 9 is 16425.

19. In an A.P., if the first term is 22, the common difference is — 4 and the sum to n terms is 64, find n.

Sol:

Given, $a = 22, d = -4, S_n = 64$

$$S_n = \frac{n}{2} (2a + (n - 1)d)$$

$$64 = \frac{n}{2} \times (2 \cdot 22 + (n - 1) \cdot (-4))$$

$$64 = n(24 - 2n)$$

$$64 = 2n(12 - n)$$

$$12n - n^2 = \frac{64}{2} = 32$$

$$n^2 - 12n + 32 = 0$$

$$(n - 4)(n - 8) = 0$$

$$\therefore n = 4 \text{ or } 8$$

20. In an A.P., if the 5th and 12th terms are 30 and 65 respectively, what is the sum of first 20 terms?

Sol:

$$\text{Given, } a_5 = 30, a_{12} = 65$$

$$\Rightarrow 30 = a + (5 - 1)d$$

$$30 = a + 4d \dots\dots(i)$$

$$\Rightarrow 65 = a + (12 - 1)d$$

$$65 = a + 11d \dots\dots(ii)$$

$$(ii) - (i) \Rightarrow a + 11d = 65$$

$$\underline{a + 4d = 30}$$

$$0 + 7d = 35$$

$$d = \frac{35}{7} = 5$$

$$\text{put } d = 5 \text{ in } \dots(i) \Rightarrow 80 = a + 4(5)$$

$$a = 80 - 20 = 60$$

$$S_{20} = \frac{20}{2}(2(60) + (20 - 1)5) \quad \left(\because S_n = \frac{n}{2}(2a + (n - 1)d) \right)$$

$$= 10[20 + 95]$$

$$= 10 \times 115$$

$$= 1150$$

$$\therefore \text{Sum of first 20 terms } S_{20} = 1150$$

21. Find the sum of the first

(i) 11 terms of the A.P : 2, 6, 10, 14, ...

(ii) 13 terms of the A.P : — 6, 0, 6, 12,....

(iii) 51 terms of the A.P : whose second term is 2 and fourth term is 8.

Sol:

(i)

$$\text{Given AP, } 2; 6, 10, 14, \dots\dots$$

$$a = 2, d = 4, S_n = S_{11} = \frac{11}{2}(2.2 + (11 - 1).4) \quad \left(\because S_n = \frac{n}{2}(2a + (n - 1)d) \right)$$

$$= \frac{11}{2}(4 + 40)$$

$$= \frac{11}{2} \times 44$$

$$\therefore S_{11} = 242$$

(ii)

$$\text{Given AP } -6, 0, 6, 12, \dots\dots$$

$$\begin{aligned}
 a &= -6, d = 6, S_n = \frac{n}{2}(2a + (n-1)d) \\
 S_n &= S_{13} = \frac{13}{2}(2 \times -6 + (13-1) \times 6) \\
 &= \frac{13}{2}(-12 + 72) \\
 &= \frac{13}{2} \times 60 \\
 &= 390 \\
 \therefore S_{13} &= 890
 \end{aligned}$$

(iii)

Given, $a_2 = 2$ and $a_4 = 8$

$$a + d = 2 \dots (i) \quad a + 3d = 8 \dots (ii)$$

$$(ii) - (i) \Rightarrow a + 3d = 8$$

$$\underline{a + d = 2}$$

$$2d = 6$$

$$d = 3$$

$$\text{put } d = 3 \text{ in } \dots (i) \Rightarrow a + d = 2$$

$$a + 3 = 2$$

$$a = -1$$

$$\begin{aligned}
 S_{51} &= \frac{51}{2}(2 \times -1 + (51-1) \times 3) \quad \left(\because S_n = \frac{n}{2}(2a + (n-1)d) \right) \\
 &= \frac{51}{2}(-2 + 50 \times 3) \\
 &= \frac{51}{2} \times 148 \\
 &= 3774. \\
 \therefore S_n &= 3774
 \end{aligned}$$

22. Find the sum of

- (i) the first 15 multiples of 8
- (ii) the first 40 positive integers divisible by (a) 3 (b) 5 (c) 6.
- (iii) all 3 — digit natural numbers which are divisible by 13.
- (iv) all 3-digit natural numbers, which are multiples of 11.

Sol:

The first 15 multiples of 8 are 8, 16, 24,

$$a = 8, d = 8, n = 15$$

$$\begin{aligned}
 S_{15} &= \frac{15}{2}(2 \times 8 + (15-1) \times 8) \quad \left(\because S_n = \frac{n}{2}(2a + (n-1)d) \right) \\
 &= \frac{15}{2}(16 + 112) \\
 &= \frac{15}{2} \times 128 \\
 &= 960
 \end{aligned}$$

$\therefore$ Sum of first 15 multiples of 8 is 960.

Given, $a_2 = 2$ and $a_4 = 8$

$$a + d = 2 \dots (i) \quad a + 3d = 8 \dots (ii)$$

$$(ii) - (i) \Rightarrow a + 3d = 8$$

$$\underline{a + d = 2}$$

$$2d = 6$$

$$d = 3$$

$$\text{Put } d = 3 \text{ in } \dots (i) \Rightarrow a + d = 2$$

$$a + 3 = 2$$

$$a = -1$$

$$S_{51} = \frac{51}{2} (2 \times -1 + (51 - 1 \times 3)) \quad \left(\because S_n = \frac{n}{2} (2a + (n - 1)d) \right)$$

$$= \frac{51}{2} (-2 + 50 \times 3)$$

$$= \frac{51}{2} \times 148$$

$$= 3774$$

$$= 44550$$

$\therefore$ Sum of all 3 – digit natural numbers which are multiples of 11 is 44550.

23. Find the sum:

$$(i) \quad 2 + 4 + 6 + \dots + 200$$

$$(ii) \quad 3 + 11 + 19 + \dots + 803$$

$$(iii) \quad 34 + 32 + 30 + \dots + 10$$

$$(iv) \quad 25 + 28 + 31 + \dots + 100$$

Sol:

$$(i) \quad 2 + 4 + 6 + \dots + 200$$

$$a = 2, d = 4 - 2 = 2, l = 200 = a_n$$

$$\therefore S_n = \frac{n}{2} (a + l) \text{ and } a_n = a + (n - 1)d$$

$$200 = 2 + (n - 1)2$$

$$198 = (n - 1)2$$

$$n - 1 = \frac{198}{2} = 99$$

$$n = 100$$

$$S_n = \frac{100}{2} (2 + 200)$$

$$= 50 \times 202$$

$$= 10100$$

$$(ii) \quad 3 + 11 + 19 + \dots + 803$$

$$a = 3, d = 11 - 3 = 8, l = a_n = 803$$

$$803 = 3 + (n - 1)8$$

$$\frac{800}{8} = n - 1$$

$$n = 101$$

$$S_n = \frac{101}{2}(3 + 803)$$

$$= \frac{101}{2} \times 806$$

$$= 504$$

$$S_n = 504$$

$$(iii) \quad 34 + 32 + 30 + \dots + 10$$

$$a = 34, d = -2, l = a_n = 10$$

$$10 = 34 + (n - 1) \times 2$$

$$+24 = 2(n - 1)$$

$$n - 1 = 12$$

$$n = 13$$

$$\therefore S_{13} = \frac{13}{2}(34 + 10)$$

$$= \frac{13}{2} \times 44$$

$$= 286$$

$$(iv) \quad 25 + 28 + 31 + \dots + 100$$

$$a = 25, d = 8, l = a_n = 100$$

$$100 = 25 + (n - 1) \times 3$$

$$75 = (n - 1) \times 3$$

$$n - 1 = 25$$

$$n = 26$$

24. Find the sum of the first 15 terms of each of the following sequences having n^{th} term as

$$(i) \quad a_n = 3 + 4n$$

$$(ii) \quad b_n = 5 + 2n$$

$$(iii) \quad Y_n = 9 - 5n$$

Sol:

$$(i) \quad \text{Given } a_n = 3 + 4n$$

$$\text{Put } n = 1, a_1 = 3 + 4(1) = 7$$

$$\text{Put } n = 15, a_{15} = 3 + 4(15) = 63 = l$$

$$\text{Sum of 15 terms, } S_{15} = \frac{15}{2}(7 + 63) \quad \left(\because S_n = \frac{n}{2}(a + l) \right)$$

$$= \frac{15}{2} \times 70$$

$$\therefore S_{15} = 525$$

$$(ii) \quad \text{Given } b_n = 5 + 2n$$

$$\text{Put } n = 1, b_1 = 5 + 2(1) = 7$$

$$\text{Put } n = 15, b_{15} = 5 + 2(15) = 35 = l$$

$$\text{Sum of 15 terms, } S_{15} = \frac{15}{2}(7 + 35) \quad \left(\because S_n = \frac{n}{2}(a + l) \right)$$

$$= \frac{15}{2} \times 42$$

$$= 315$$

$$\therefore S_{15} = 315$$

(iii) Given, $Y_n = 9 - 5n$

$$\text{Put } n = 1, y_1 = 9 - 5.1 = -4$$

$$\text{Put } n = 15, y_{15} = 9 - 5.15 = 9 - 75 = -66 = (l)$$

$$\therefore S_{15} = \frac{15}{2}(-4 - 66) \quad \left(\because S_n = \frac{n}{2}(a + l) \right)$$

$$= \frac{15}{2} \times -70$$

$$= -465$$

$$\therefore S_{15} = -465$$

25. Find the sum of first 20 terms of the sequence whose n^{th} term is $a = An + B$.

Sol:

$$\text{Given, } n^{\text{th}} \text{ term } a_n = An + B$$

$$\text{Put } n = 1, a_1 = A + B$$

$$\text{Put } n = 20, a_{20} = 20A + B = (l)$$

$$\therefore S_{20} = \frac{20}{2}(A + B + 20A + B) \quad \left(\because S_n = \frac{n}{2}(a + l) \right)$$

$$= 10(21A + 2B)$$

$$= 210A + 20B$$

$$\therefore S_n = 210A + 20B$$

26. Find the sum of the first 25 terms of an A.P. whose n^{th} term is given by $a_n = 2 - 3n$.

Sol:

$$\text{Given, } n^{\text{th}} \text{ term } a_n = 2 - 3n$$

$$\text{Put } n = 1, a_1 = 2 - 3.1 = -1$$

$$\text{Put } n = 25, a_{25} = l = 2 - 3.25 = -43$$

$$\therefore S_{25} = \frac{25}{2}(-1 - 43) = \frac{25}{2}(-44) = -925$$

$$\therefore S_{25} = -925$$

27. Find the sum of the first 25 terms of an A.P. whose n^{th} term is given by $a_n = 7 - 3n$.

Sol:

$$\text{Given, } a_n = 7 - 3n$$

$$\text{Put } n = 1, a_1 = 7 - 3.1 = 4$$

$$\text{Put } n = 25, a_{25} = l = 7 - 3.25 = -68$$

$$\therefore S_{25} = \frac{25}{2}(4 - 68) \quad \left(\because S_n = \frac{n}{2}(a + l) \right)$$

$$= \frac{25}{2} \times -64$$

$$= -800$$

$$\therefore S_{25} = -800$$

28. Find the sum of first 51 terms of an A.P. whose second and third terms are 14 and 18 respectively.

Sol:

$$\text{Given, } a_2 = 14 \Rightarrow a + d = 14 \dots (i)$$

$$a_3 = 18 \Rightarrow a + 2d = 18 \dots (ii)$$

$$(ii) - (i) \Rightarrow a + 2d = 18$$

$$\underline{a + d = 14}$$

$$0 + d = 4$$

$$\text{Put } d = 4 \text{ in (i) } a + 4 = 14$$

$$a = 10$$

$$\therefore S_{51} = \frac{51}{2} \{2 \cdot 10 + (51 - 1) \times 4\} \quad \left(S_n = \frac{n}{2} \{2a + (n - 1)d\} \right)$$

$$= \frac{51}{2} \{20 + 200\}$$

$$= \frac{51}{2} \times 220$$

$$= 5610$$

$$\therefore S_{51} = 5610$$

29. If the sum of 7 terms of an A.P. is 49 and that of 17 terms is 289, find the sum of n terms.

Sol:

$$\text{Given, } S_7 = 49$$

$$\frac{7}{2} (2a + (7 - 1)d) = 49 \quad \left(\therefore S_n = \frac{n}{2} \{2a + (n - 1)d\} \right)$$

$$\frac{7}{2} (2a + 6d) = 49$$

$$\frac{7}{2} \times 2(a + 3d) = 49$$

$$a + 3d = \frac{49}{7} = 7 \dots (i) \text{ and}$$

$$S_{17} = 289$$

$$\frac{17}{2} (2a + (17 - 1)d) = 289$$

$$\frac{17}{2} \times 2(a + 8d) = 289$$

$$a + 8d = \frac{289}{17} = 17 \dots (ii)$$

Subtract (i) from (ii)

$$a + 8d = 17$$

$$\underline{a + 3d = 7}$$

$$5d = 10$$

$$d = 2$$

$$\text{put } d = 2, \text{ in (i) } \Rightarrow a + 3 \times 2 = 7$$

$$a = 1$$

$$\therefore S_n = \frac{n}{2} \{2 \cdot 1 + (n - 1) \cdot 2\} \quad \left(\therefore S_n = \frac{n}{2} (2a + (n - 1)d) \right)$$

$$= n\{1 + n - 1\}$$

$$\therefore S_n = n^2.$$

30. The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Sol:

Given, $a = 5$, $l = 45$, Sum of terms = 400

$$\therefore S_n = 400$$

$$\frac{n}{2}\{5 + 45\} = 400$$

$$\frac{n}{2} = 50 = 400$$

$$n = 40 \times \frac{2}{5}$$

$$\therefore n = 16$$

16th term is 45

$$a_{16} = 45 \Rightarrow 5 + (16 - 1) \times d = 45 = 15 \times d = 40$$

$$d = \frac{40}{15} = \frac{8}{3}$$

$$\therefore n = 16, d = \frac{8}{3}$$

31. In an A.P., the sum of first n terms is $\frac{3n^2}{2} + \frac{13}{2}n$. Find its 25th term.

Sol:

$$\text{Given, sum of } n \text{ terms } S_n = \frac{3n^2}{2} + \frac{13}{2}n$$

$$\text{Let, } a_n = S_n - S_{n-1} \quad (\because \text{Replace } n \text{ by } (n-1) \text{ is } S_n \text{ to get } S_{n-1} = \frac{3(n-1)^2}{2} + \frac{13}{2}(n-1))$$

$$a_n = \frac{3n^2}{2} + \frac{13}{2}n - \frac{3(n-1)^2}{2} - \frac{13}{2}(n-1)$$

$$= \frac{3}{2}\{n^2 - (n-1)^2\} + \frac{13}{2}\{n - (n-1)\}$$

$$= \frac{3}{2}\{n^2 - n^2 + 2n - 1\} + \frac{13}{2}\{1\}$$

$$= 3n + \frac{10}{2} = 3n + 5$$

$$\text{Put } n = 25, a_{25} = 3(25) + 5 = 75 + 5 = 80$$

$$\therefore 25^{\text{th}} \text{ term } a_{25} = 80$$

32. Let there be an A.P. with first term 'a', common difference 'd'. If a_n denotes its n th term and S_n the sum of first n terms, find.

(i) n and S_n and if $a = 5$, $d = 3$ and $a = 50$

(ii) n and a , if $a_n = 4$, $d = 2$ and $S_n = -14$

(iii) d , if $a = 3$, $n = 8$ and $S_n = 192$

(iv) a , if $a_n = 28$, $S_n = 144$ and $n = 9$

(v) n and d , if $a = 8$, $a = 62$ and $S_n = 210$

(vi) n and a_n , if $a = 2$, $d = 8$ and $S_n = 90$

Sol:

- (i) Given
- $a = 5$
- ,
- $d = 3$
- ,
- $a_n = 50$

$$a_n = 50$$

$$a + (n - 1)d = 50$$

$$5 + (n - 1)3 = 50$$

$$(n - 1)3 = 45$$

$$n - 1 = \frac{45}{3} = 15$$

$$n = 16$$

$$\text{Sum of } n \text{ terms } S_n = \frac{n}{2}[a + l]$$

$$= \frac{16}{2}[5 + 50]$$

$$= 8 \times 55$$

$$= 440$$

- (ii) Given,
- $a_n = 4$
- ,
- $d = 2$
- ,
- $S_n = -14$

$$a + (n - 1) \cdot 2 = 4 \text{ and } \frac{n}{2}[2a + (n - 1) \cdot 2] = -14$$

$$a + 2n = 6 \qquad n[2a + 2n - 2] = -14$$

(or)

$$\frac{n}{2}[a + a_n] = -14$$

$$\frac{n}{2}[a + 4] = -14$$

$$n[6 - 2n + 4] = -28$$

$$n[10 - 2n] = -28$$

$$2n^2 - 10n - 28 = 0$$

$$2(n^2 - 5n - 14) = 0$$

$$(n + 2)(n - 7) = 0$$

$$n = -2, n = 7$$

 $\therefore n = -2$ is not a natural number. So, $n = 7$.

- (iii) Given,
- $a = 3$
- ,
- $n = 8$
- ,
- $S_n = 192$
- .

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$192 \times 2 = 8[6 + (8 - 1)d]$$

$$\frac{192 \times 2}{8} = 6 + 7d$$

$$48 = 6 + 7d$$

$$7d = 42$$

$$d = 6$$

- (iv) Given,
- $a_n = 28$
- ,
- $S_n = 144$
- ,
- $n = 9$

$$S_n = \frac{n}{2}[a + l]$$

$$144 = \frac{9}{2}[a + 28]$$

$$144 = \frac{9}{2}a + 126$$

$$a + 28 = 32$$

$$a = 4$$

(v) Given, $a = 8$, 62 and $S_n = 210$

$$S_n = \frac{n}{2}[a + l]$$

$$210 = \frac{n}{2}[8 + 62]$$

$$210 \times 2 = n[70]$$

$$n = \frac{210 \times 2}{70} = 6$$

$$a + (n - 1)d = 62$$

$$8 + (6 - 1)d = 62$$

$$5d = 54$$

$$d = 10.8$$

(vi) Given

$$a = 2, d = 8 \text{ and } S_n = 90$$

$$90 = \frac{n}{2}[4 + (n - 1)8] \quad \left(\because S_n = \frac{n}{2}[2a + (n - 1)d] \right)$$

$$180 = n[4 + 8n - 8]$$

$$8n^2 - 4n - 180 = 0$$

$$4(2n^2 - n - 45) = 0$$

$$2n^2 - n - 45 = 0$$

$$(2n + 1)(n - 5) = 0$$

$$\because n = -\frac{1}{2} \text{ is not a natural no. } n = 5$$

$$a_n = 2 + 4(8) \quad (\because a_n = a + (n - 1)d)$$

$$a_n = 32$$

33. A man saved Rs 16500 in ten years. In each year after the first he saved Rs 100 more than he did in the preceding year. How much did he save in the first year?

Sol:

Let 'a' be the money he saved in first year

$\Rightarrow$ First year he saved the money = Rs a

He saved Rs 100 more than, he did in preceding year.

$\Rightarrow$ Second year he saved the money = Rs (a + 100)

$\Rightarrow$ Third year he saved the money = Rs. (a + 2 (100))

So, the sequence is a, a + 100, a + 2(100),, This is in AP with common difference (d) = 100.

$\Rightarrow$ Sum of money he saved in 10 years $S_{10} = 16,500 \text{ rupees}$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$S_{10} = \frac{10}{2}(2a + (10 - 1) \cdot 100)$$

$$16,500 = 5(2a + 9 \times 100)$$

$$2a + 900 = \frac{16500}{5} = 3300$$

$$2a = 2400$$

$$a = \frac{2400}{2} = 1200$$

$\therefore$ He saved the money in first year (a) = Rs. 1200

34. A man saved Rs 32 during the first year, Rs 36 in the second year and in this way he increases his savings by Rs 4 every year. Find in what time his saving will be Rs 200.

Sol:

Given

Saving in 1st yr (a_1) = Rs 32

Saving in 2nd yr (a_2) = Rs 36

Increase in salary every year (d) = Rs 4

Let in n years his saving will be Rs 200

$$\Rightarrow S_n = 200$$

$$\Rightarrow \frac{n}{2} [2a + (n - 1)d] = 200$$

$$\Rightarrow \frac{n}{2} [64 + 4n - 4] = 200$$

$$\Rightarrow \frac{n}{2} [4n + 60] = 200$$

$$\Rightarrow 2n^2 + 30n = 200$$

$$\Rightarrow n^2 + 15n - 100 = 0 \quad [\text{Divide by 2}]$$

$$\Rightarrow n^2 + 20n - 5n - 100 = 0$$

$$\Rightarrow n(n + 20) - 5(n + 20) = 0$$

$$\Rightarrow (n + 20)(n - 5) = 0$$

$$\text{If } n + 20 = 0 \text{ or } n - 5 = 0$$

$$n = -20 \quad \text{or } n = 5 \quad (\text{Rejected as } n \text{ cannot be negative})$$

$\therefore$ In 5 years his saving will be Rs 200

35. A man arranges to pay off a debt of Rs 3600 by 40 annual installments which form an arithmetic series. When 30 of the installments are paid, he dies leaving one-third of the debt unpaid, find the value of the first installment.

Sol:

Given

A man arranges to pay off a debt of Rs 3600 by 40 annual installments which form an A.P
i.e., sum of all 40 installments = Rs 3600

$$S_{40} = 3600$$

Let, the money he paid in first installment is a, and every year he paid with common difference = d

Then,

$$S_{40} = 3600 \quad (\because S_n = \frac{n}{2} [2a + (n - 1)d])$$

$$\frac{40}{x} [2a + (40 - 1)d] = 3600$$

$$2a + 39d = \frac{3600}{20} = 180 \dots \dots (i)$$

but,

He died by leaving one third of the debt unpaid that means he paid remaining money in 30 installments.

$$\therefore \text{The money he paid in 30 installments} = 3600 - \frac{3600}{3} = 3600 - 1200$$

$$\therefore S_{30} = 2400$$

$$S_{30} = 2400$$

$$= \frac{30}{2} [2a + (30 - 1)d] = 2400 \quad \left(\because S_n = \frac{n}{2} (2a + (n - 1)d) \right)$$

$$2a + 29d = \frac{2400}{15} = 160 \dots \dots (ii)$$

$$(i) - (ii) \Rightarrow 2a + 39d = 180$$

$$\underline{2a + 29d = 160}$$

$$0 + 10d = 20$$

$$d = \frac{20}{10} = 2$$

$$\text{put } d = 2 \text{ in (ii) } 2a + 29(2) = 160$$

$$2a = 102$$

$$a = \frac{102}{2} = 51$$

$\therefore$ The value of his first installment = 51.

Exercise – 9.1